TOPIC 1 NUMBER AND ALGEBRA

Q1. a) 8.235×10^{-16}

b) $0.0649484848\ldots \approx 6.495 \times 10^{-2}$

c) 3.634×10^{-12}

Q2. 1 gram $\rightarrow$ 6.022×10^{23} hydrogen atoms } given information

Sun: 1.99×10^{30} kg 74.9% hydrogen }

Find how much hydrogen (in kg) Sun contains.

74.9% of 1.99×10^{30} is:

$0.749 \times 1.99 \times 10^{30} = 1.4905 \times 10^{30}$ kg of hydrogen

Convert kg $\rightarrow$ grams (1 kg = 1000 g)

$1.4905 \times 10^{30} \times 1000 = 1.4905 \times 10^{33}$ grams

$1.4905 \times 10^{33} \times 6.022 \times 10^{23} = 8.98 \times 10^{56}$ hydrogen atoms

Q3. a) $pH = -\log_{10}(5.2 \times 10^{-8}) = 7.2839966\ldots$

$\approx 7.28\ (Me^{-1})$

b) $4.2 = -\log_{10} C$ (NSolve, you need to add , x, 0)

$C = 6.31 \times 10^{-5}$

Q4. a) arithmetic sequence

$d = 115 - 85 = 30 \quad u_1 = 85$

$u_{15} = 85 + (15-1) \times 30 = 505$

b) $S_{15} = \frac{15}{2}(2 \times 85 + (15-1) \times 30) = 4425$

c) mean $= \frac{\text{Total number of plants}}{\text{months}} = \frac{4425}{15} = 295$

Q5. 2.4% of 300 is: $0.024 \times 300 = \$7.2$ per year

$d = 7.2 \quad u_1 = 300$

300 u_1 → after 1 year → u_2 → after 2 years → u_3 .... → after 10 years → u_{11}

$u_{11} = 300 + (11-1) \times 7.2 = \$372$

Q6. $S_n = \sum_{r=1}^{n} (5r+11)$

First term $u_1 = S_1$

$u_1 = S_1 = \sum_{r=1}^{1} (5r+11) = 16$

$u_1 + u_2 = S_2 = \sum_{r=1}^{2} (5r+11) = 37$

$u_1 + u_2 = 37 \Rightarrow u_2 = 37 - 16 = 21$

$d = u_2 - u_1 = 21 - 16 = 5$

$u_1 = 16$

$d = 5$

Q7. a) $u_1 = 12$

$u_2 = 23$

$d = 23 - 12 = 11$

$221 = 12 + (n-1) \times 11$ (NSolve)

$n = 20$

b) $u_{18} = 12 + 17 \times 11 = 199$

$u_{19} = 210$

$u_{20} = 221$ Total number of edges:

$199 + 210 + 221 = 630$

Q8. a) $u_1 = 1\,km = 1000\,m$

$d = 300\,m$

$u_5 = 1000 + (5-1)\times 300 = 2200\,m = 2.2\,km$

b) i) $u_n = 10\,km = 10\,000\,m$

$10\,000 = 1000 + (n-1)\times 300$ (NSolve)

$n = 31$ days

ii) $S_{31} = \frac{31}{2}(2\times 1000 + (31-1)\times 300)$

$= 170\,500\,m = 170.5\,km$

Q9. a) $u_1 = 25 \quad d = 1.6$

$u_{10} = 25 + (10-1)\times 1.6 = 39.4$ sec

b) $u_n = 55.4$

$55.4 = 25 + (n-1)\times 1.6$ (NSolve)

$n = 20$ laps

c) $S_{20} = \frac{20}{2}(2\times 25 + (20-1)\times 1.6) = 804$ seconds

1 minute $\rightarrow$ 60 seconds

? $\leftarrow$ 804 seconds

$804 \div 60 = 13.4$ minutes

$0.4 \times 60 = 24$ seconds

Total time 13min 24 seconds

Q10. a) $r = \frac{256}{1024} = \frac{1}{4} = 0.25$

b) $u_8 = 1024 \times (0.25)^{8-1} = 0.0625$

c) $S_{12} = \frac{1024(0.25^{12}-1)}{0.25-1} = 1365.33\ldots = 1365$

Q11. growth by 20% → $r = 1.2$

a) $u_1 = 50000 \quad r = 1.2$

$u_{12} = 50000 \times 1.2^{12-1} = 371504.185\ldots = 372000$

b) because bacteria consume resources in the petri dish, which will eventually run out.

Q12. $u_1 = S_1$

$u_1 = \sum_{r=1}^{1} 2 \times 5^r = 10$

$u_1 + u_2 = S_2 = \sum_{r=1}^{2} 2 \times 5^r = 60$

$u_1 + u_2 = 60 \quad u_2 = 60 - 10 = 50$

$r = \frac{50}{10} = 5$

Q13. a) $n = 10 \quad r = 0.98 \quad u_1 = 68000$

$u_g = 68000 \times 0.98^{10-1} = 56\,694.847\ldots$

b) $50\,000 = 68000 \times 0.98^n$

$n = 16.22$

$u_{16} > 50\,000 \quad u_{17} < 50\,000$

$u_{16} = 2036 \quad u_{17} = 2037$

Population of kiwis will fall below 50000 in 2036

Q14. a) $u_1 = 100 \quad d = 15$

$u_8 = 100 + (8-1) \times 15 = 205$

b) $S_{12} = \frac{12}{2}(2 \times 100 + (12-1) \times 15) = 2190$ km

c) $2500 = \frac{12}{2}(2 \times 100 + (12-1) \times k)$

$k = 19.697 \quad k = 20$

d) $u_1 = 110 \qquad r = 0.98$

$u_{12} = 110 \times 0.98^{12-1} = 88.0804 = 88.1$ sec.

e) $u_1 = 110 \qquad u_{12} = 85 \qquad n = 12$

$85 = 110 \times r^{12-1} \qquad r = 0.976834\ldots$

$m = 2.32\ \%$

Q15. a) $u_1 = 10 \qquad d = 6$

$u_{12} = 10 + (12-1) \times 6 = 76$

b) $S_{15} = \frac{15}{2}(2 \times 10 + (15-1) \times 6) = 480$

c) $u_1 = 10 \qquad d = x \qquad u_{15} = 1000$

$1000 = \frac{15}{2}(2 \times 10 + 14 \times x) \qquad x = 8.09524$

$x = 9$

d) $u_1 = 17.1 \qquad r = 0.95$

$u_5 = 17.1 \times 0.95^{5-1} = 13.9281 = 13.9$

e) $S_{16} = \frac{17.1(0.95^{16} - 1)}{0.95 - 1} = 191.476\ldots = 191$

f) $u_0 = 17.1 \times 0.95^{0-1} = 18$

Q16. $\sum_{n=1}^{6}(5n-2) = (5\times1-2) + (5\times2-2) + (5\times3-2) + (5\times4-2) +$ or GDC
$+ (5\times5-2) + (5\times6-2) = 93$

Q17. a) $PV = -24000 \qquad n = 5 \qquad I = 6\% \qquad cpy = 1$

$FV = 32\,117$

b) $PV = ? \qquad 23\,898$

$FV = 32\,117$

$n = 5 \qquad I\% = 6 \qquad cpy = 2$

Q18. a) PV = −2500 n = 1

I = 6.2

cpy = 12

FV = 2659.48

b) I% = ? n = 1

cpy = 1

PV = −2500

FV = 2659.49 ← FV has to be more!

I = 6.38%

Q19. a) PV = −24900

I% = −16

n = 10

cpy = 1

FV = 43355.04

b) PV = −12000

n = 6

FV = 6200

cpy = 1

I% = 10.4%

Q20. a) what amount of money, in 8 years, would have the same value as $5000 today

n = 8

I% = 3%

PV = −5000 FV = 6333.85

cpy = 1

Now we find PV to ensure he has 6333.85 at the end of 8 years

n = 8 I% = 4.1 FV = 6333.85 PV = 4592.64

cpy = 1

b) $n = 8$ $PV = -2000$ $Py = 1$

$1\% = 4.1$ $FV = 6333.85$ $Cy = 1$

$Pmt = ?$ $Pmt = 386.67$

Q21. a) 16 c) $x = \log_{10} k$

b) $8x^3$ d) $2x - 6 = \ln_e 5$

$2x = \ln 5 + 6$ $x = \frac{\ln 5 + 6}{2}$

Q22. a) $b = 0.0525$

$m = 1 - 2.5 \log_{10}(0.0525) = 4.1996$

b) $7 = 1 - 2.5 \log_{10}(b)$

$b = 0.003981$

c) $0.0525 \div 0.003981 = 13.2$ times

Q23. $8342 \approx 8300$ $2.568 \approx 2.6$ $0.0134 \approx 0.013$

$1.086 \approx 1.09$

Q24. a) 26.67 b) 5.93

Q25. lower bound 385 upper bound 395

Q26. lower bound 57.65 upper bound 57.75

Q27. a) $80cm = 0.8m$ $19 \times 0.8 = 15.2$ m

b) $a = 0.8 \times 18.5 = 14.8$ m

$b = 0.8 \times 19.5 = 15.6$ m

c) $\left| \frac{15.2 - 14.8}{14.8} \right| \times 100 = 2.70\%$

Q28. a) 3 hours 47 min = 3.78 h

$400 \div 3.78 = 105.820 \approx 106$ km/h

b) Exact value is 3.78 h

Approximate value is $400 \div 106 = 3.77358...$

$$\left| \frac{3.77358 - 3.78}{3.78} \right| \times 100\% = 0.169841...$$

$$= 0.170\ \%$$

Q29. a) PV = −800

1% = 7.5 % cpy = 12

pmt = −500 ppy = 12

FV = 10 000 n = ? n = 17.307...

k = 18 months

b) Total deposit is

800 + 500 × 18 = 9800

n = 18
PV = −800
1% = 7.5 % cpy = 12
pmt = −500 ppy = 12 FV = 10 389.39

10 389.39 − 9800 = 589.39

Q30. a) i) 25 % of 285 000 is ¥1250

285 000 − ¥1250 = ¥213 750

ii) n = 60 FV = 0
1% = 4.5
PV = 213 750 Pmt = ? 3984.95
py = 12
cy = 12

b) $3984.95 \times 60 = 239097$

c) $I\% = 4.5$ $PV = 213750$ $Pmt = -4600$ $FV = 0$ $Py = 12$ $Cy = 12$ $N = ?$ $= 52$

d) $n = 52$ $I\% = 4.5$ $PV = 213750$ $Pmt = -4600$ $Py = 12$ $Cy = 12$ $FV = ?$ $FV = 3893.20$

$4600 - 3893.20 = 706.80$

e) $51 \times 4600 + 706.80 = 235306.80$

$239097 - 235306.80 = 3790$

Q31. a) $n = 2$ $PV = -37000$ $I\% = 6.4$ $Py = 1$ $Cy = 4$ $FV = 42010$

b) $PV = -37000$ $FV = 50000$ $I\% = 6.4\%$ $Py = 1$ $Cy = 4$

$n = 4.74230...$

$n = 4.74$ years

$m = 57$ months

c) 25% of 200 000 is 50 000

Amount of the loan $200000 - 50000 =$

$= 150000$

d) i) $120 \times 1700 - 150000 = 54000$

ii) $n = 120$ $PV = 150000$ $pmt = -1700$ $FV = 0$ $Py = 12$ $Cy = 12$

$r = 6.45779...$

$= 6.46\%$

e) $n = 60$
$I = 6.46$
$PV = 150\,000$ $\quad FV = 86973$
$pmt = -1700$
$py = 12$
$cy = 12$

f) $204\,000 - (60 \times 1700 + 86973)$

$= 15\,027$