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APPLICATIONS AND INTERPRETATIONS (AI)

STANDARD LEVEL (SL)

REVISION

TOPIC1: NUMBER AND ALGEBRA

1.1	
Operations with numbers in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.	Calculator or computer notation is not acceptable. For example, 5.2E30 is not acceptable and should be written as 5.2×10^{30}.

Q1. Express the following in scientific notation correct to 4 significant figures.

a) $(1 - 0.993)^7$

b) $\frac{1}{\sqrt[3]{3650}}$

c) $\left(\frac{3}{56}\right)^9$

Q2. One gram of hydrogen contains approximately 6.022×10^{23} hydrogen atoms. The Sun weighs approximately 1.99×10^{30} kg and is 74.9% hydrogen. Estimate the number of hydrogen atoms in the Sun.

Q3. The pH of a solution is given by the formula $pH = -\log_{10} C$ where C is the hydrogen ion concentration in a solution, measured in moles per litre (Ml^{-1}).

a) Find the pH value for a solution in which the hydrogen ion concentration is 5.2×10^{-8}.
b) Find the hydrogen ion concentration in a solution with pH 4.2. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

1.2	
Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways. If technology is used in examinations, students will be expected to identify the first term and the common difference.
Applications.	Examples include simple interest over a number of years.
Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.	Students will need to approximate common differences.

FORMULA

The nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right)\ ;\ S_n = \frac{n}{2}(u_1 + u_n)$

Q4. In the first month of a reforestation program, the town of Neerim plants trees. Each subsequent month the number of trees planted will increase by an additional trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

a) Find the number of trees to be planted in the 15th month.
b) Find the total number of trees to be planted in the first 15 months.
c) Find the mean number of trees planted per month during the first 15 months.

Q5. Juanita invests $300 at 2.4% simple interest. How much will be in her account after 10 years?

Q6. Determine the first term and common difference of an arithmetic sequence where the sum of the first n terms is given by $S_n = \sum_{r=1}^{n}(5r + 11)$.

Q7. Consider the following sequence of figures.

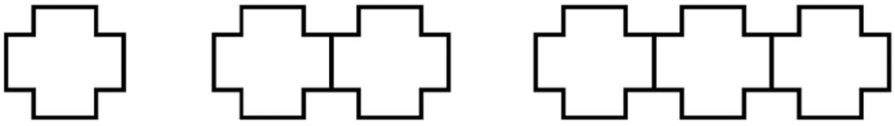


Figure 1 Figure 2 Figure 3

Figure 1 contains 12 edges and Figure 2 contains 23 edges.

(a) Given that Figure n contains 221 edges, find n.
(b) Find the total number of edges of figure 18, Figure 19 and Figure 20.

Q8. Sam enters a 10 km running race. On the first day of training, she runs 1 km. On each subsequent day she runs 300 m more than the previous day.

(a) Find how far she runs on the fifth day of training.

On her final day of training, she runs 10 km.

(b) Find
(i) the total number of days she spends training.
(ii) the total distance she runs during training.

Q9. On Wednesday Eddy goes to a velodrome to train. He cycles the first lap of the track in 25 seconds. Each lap Eddy cycles takes him 1.6 seconds longer than the previous lap.

(a) Find the time, in seconds, Eddy takes to cycle his tenth lap.

Eddy cycles his last lap in 55.4 seconds.

(b) Find how many laps he has cycled on Wednesday.
(c) Find the total time, in minutes and seconds, cycled by Eddy on Wednesday.

1.3	
Geometric sequences and series. Use of the formulae for the n^{th} term and the sum of the first n terms of the sequence.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways.
Use of sigma notation for the sums of geometric sequences.	If technology is used in examinations, students will be expected to identify the first term and the ratio.
Applications.	Examples include the spread of disease, salary increase and decrease and population growth.

FORMULA

The nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \ r \neq 1$

Q10. The first three terms of a geometric sequence are 1024, 256 and 64.

(a) Write down the value of r, the common ratio of the geometric sequence.

(b) Find u_8.

(c) Find the sum of the first twelve terms of this sequence, giving the answer correct to the nearest integer.

Q11. The population of bacteria in a petri dish grows by 20% each day. There are initially 50000 bacteria in the dish.

a) Find the number of bacteria in the dish after 12 days.

b) Explain why this model cannot continue indefinitely.

Q12. Determine the first term and common ratio of a geometric sequence where the sum of the first n terms is given by $S_n = \sum_{r=1}^{n} 2 \times 5^r$.

Q13. The kiwi is a flightless bird and is a national treasure in New Zealand. At the start of 2021 there were approximately 68 000 kiwi left, with the population decreasing by 2% every year.

(a) Find the expected population size of kiwis in 2030 assuming the rate of decrease in kiwi population remains the same.

(b) Find the year in which the population of kiwis falls below 50 000 assuming the rate of decrease in kiwi population remains the same.

Q14. Sarah has started a 12 week training proram to improve her cycling. She plans to cycle 100 kilometres in the first week and, in each subsequent weel, cycle 15 kilometres further than she did the wewek before.

(a) Calculate the distance she will cycle during the eighth week.

(b) Calculate the total distance she will have cycled by the end of the training program.

Sarah's coach tells her she should have cycled 2500 kilometres by rthe ned of the training program, so Sarah decides to increase her cycling distances. She still plans to cylce 100 kilomteres in the first week, but in each subsequent week, she will cycle k kilometres more than she did the week before.

(c) Calculate the minimum integer value of k required for Sarah to reach the target of 2500 kilometres in 12 weeks.

As part of her training, each week Sarah perfomrs a one kilometre time trial, in which she aims to cycle one kilometre as fast as she can. Sarah's time decreases by 2% each week. In the first week, her time is exactly 110 seconds.

(d) Calculate Sarah's tiem in the 12th week.

Sarah's goal is to cycle one kilometre in less than 85 seconds by the end of the tarining program. To do this, she needs to decrease her time by at least $m\%$ per week from her time of 110 seconds in week one.

(e) Find the minimum value of m so that Sarah reaches her goal.

Q15. Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

a) Calculate the number of pendulums she would make on the 12th day.

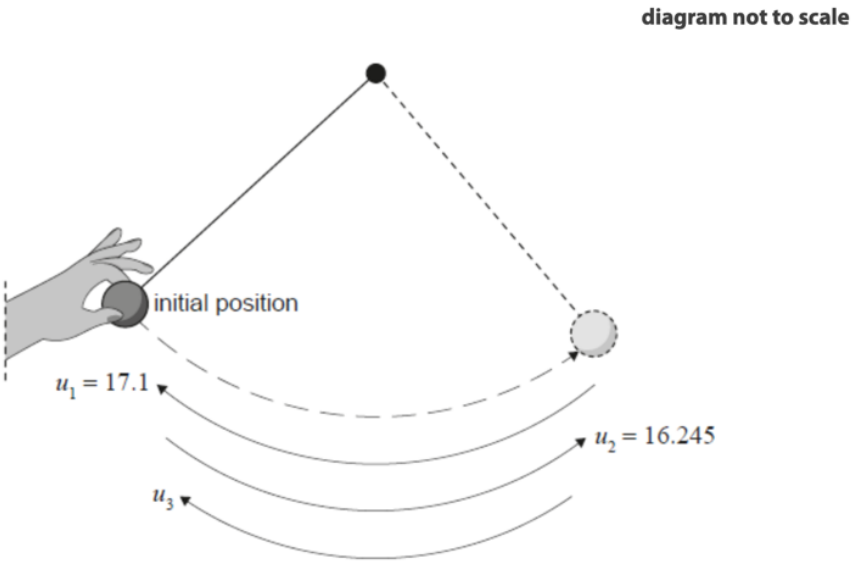
She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

b) Calculate the total number of pendulums she would have available at the market.

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

c) Given that she will still make pendulums for a total of days, calculate the minimum integer value of required for her to reach her target.

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.



On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

d) Calculate the distance that the ball travelled during the 5th recorded swing.
e) Calculate the total distance that the ball travelled during the first 16 recorded swings.
f) Calculate the distance that the ball travelled before Daina started recording.

1.4	
Financial applications of geometric sequences and series: • compound interest • annual depreciation.	Examination questions may require the use of technology, including built-in financial packages. The concept of simple interest may be used as an introduction to compound interest. Calculate the real value of an investment with an interest rate and an inflation rate. In examinations, questions that ask students to derive the formula will not be set. Compound interest can be calculated yearly, half-yearly, quarterly or monthly.

FORMULA

Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$, where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest

Q16. For this question, give all the answers correct to the nearest USD.

24000 USD is invested for 5 years at a nominal annual interest rate of 6%, compounded yearly.

a) Find the value of P, the amount of money after 5 years.

b) An amount of money Q is invested for 5 years at a nominal annual interest rate of 6%, compounded half – yearly. The amount of money after 5 years is P. Find the value of Q.

Q17. Hayley invests $2500 in an investment account pays 6.2% interest compounded monthly.

a) Find the value of the investment after 1 year.

A different investment account pays $r\%$ interest compounded annually.

b) Find the minimum value of r to 2 decimal places so that it is more beneficial for Hayley to invest her money in this account.

Q18. Hannah buys a car for $24900. The value of the car depreciates by 16% each year.

(a) Find the value of the car after 10 years.

Patrick buys a car for $12000. The car depreciates by a fixed amount each year, and after 6 years it is worth $6200.

(b) Find the annual rate of depreciation of the car.

Q19.

Mike wants to deposit part of his savings in a bank account that pays an annual interest rate of $4.1\,\%$ **compounded annually**. The annual inflation rate is expected to be 3% per year throughout the following 8 years. Mike wants his initial deposit to have a real value of \$5 000 after 8 years, compared to current values.

The bank gives Mike two proposals:

Proposal 1: A one-time investment at the start of the 8-year period.

Proposal 2: Invest \$2 000 at the start of the 8-year period and make payments of \$ x at the end of each year.

(a) Find the minimum amount Mike should deposit if he accepts proposal 1. Round your answer to the nearest dollar.

(b) Find the minimum value of the annual payments, x, if Mike accepts proposal 2. Round your answer to the nearest dollar.

1.5	
Laws of exponents with integer exponents	$5^3 \times 5^{-6} = 5^{-3}, 6^4 \div 6^3 = 6, (2^3)^{-4} = 2^{-12},$ $(2x)^4 = 16x^4, 2x^{-3} = \frac{2}{x^3}$
Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.	Awareness that $a^x = b$ is equivalent to $\log_a b = x$, that $b > 0$, and $\log_e x = \ln x$.

FORMULA

Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$, where $a > 0, b > 0, a \neq 1$

Q20.

a) Evaluate $(2^{-2})^{-2}$

b) Simplify $(2x)^3$

c) Solve $10^x = k$

d) If $e^{2x-6} = 5$ find x in terms of natural logarithms.

Q21. Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens.

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres.

(c) Find how many times brighter Acubens is compared to Ceres.

1.6	
Approximation: decimal places, significant figures.	Students should be able to choose an appropriate degree of accuracy based on given data.
Upper and lower bounds of rounded numbers	If x=4.1 to one decimal place, 4.05≤x<4.15.
Percentage errors.	Students should be aware of, and able to calculate, measurement errors (such as rounding errors or measurement limitations). For example finding the maximum percentage error in the area of a circle if the radius measured is 2.5 cm to one decimal place.
Estimation.	Students should be able to recognize whether the results of calculations are reasonable. For example lengths cannot be negative.

FORMULA BOOKLET

Percentage error	$\varepsilon = \left\| \frac{v_A - v_E}{v_E} \right\| \times 100\%$, where v_E is the exact value and v_A is the approximate value of v

Q22. Round to 2 significant figures:

8342 2.568 0.0134 1.086

Q23. Calculate the following, rounding your answers to 2 decimal places.

a) $37.4 - 16.1 \div (4.2 - 2.7)$

b) $\frac{27.9-17.3}{8.6} + 4.7$

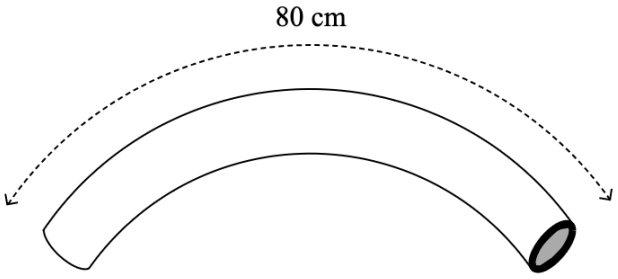
Q24. What is the upper bound and lower bound of 390 grams, measured to the nearest 10 grams?

Q25. 57.7 has been rounded to 1 decimal place. Work out the **upper** and **lower** bounds of this value.

Q26.

[Maximum points: 7]

A swimming pool has a length of 25 m. A student uses a flexible swim noodle with a length of 80 cm to measure the pool's width.



The student estimates the width of the pool to be equal to 19 noodles, rounded to the nearest whole noodle.

(a) Calculate the length of this estimate in metres. [2]

Let W represent the actual width of the pool.

(b) If $a \leq W < b$ determine the values of a and b in metres. [3]

(c) If the actual width is equal to the value of a calculated in part (b) determine the percentage error of the estimation in part (a). [2]

Q27.

It takes Drew 3 hours 47 minutes to drive 400 km.

(a) Show that his average speed is 106 km/h to the nearest integer.

Drew informs his sister of his average speed using the rounded value from part (a). His sister then uses this value to calculate how long the journey took.

(b) Calculate the percentage error in this calculation.

1.7	
Amortization and annuities using technology.	Technology includes the built-in financial packages of graphic display calculators, spreadsheets. In examinations the payments will be made at the end of the period. Knowledge of the annuity formula will enhance understanding but will not be examined.

Q28. On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional EUR 500 into the savings account.

At the end of $k$months, Laviola will have saved enough money to withdraw EUR 10000.

(a) Find the smallest possible value of k, for $k \in \mathbb{Z}^+$.

(b) For this value of k, find the interest earned in the savings account. Express your answer correct to the nearest EUR.

Q29. Ruben wants to buy a car for a price of 285 000 South African rand (ZAR). He goes to a bank to get a loan to buy the car. To be eligible for the loan, Ruben must make an initial down payment equal to 25% of the price of the car.

The bank offers him a 5-year loan for the remaining balance, with a 4.5% nominal interest rate per annum, compounded monthly. Ruben will pay the loan in fixed payments at the end of each month.

(a.i) Find the original amount of the loan after the down payment is paid. Give the exact answer.

(a.ii) Calculate Ruben's monthly payment for this loan, to two decimal places.

(b) Using your answer from part (a)(ii), calculate the total amount Ruben will pay over the life of the loan, to the nearest ZAR. Do **not** include the initial down payment.

Ruben would like to repay the loan faster and increases his payments such that he pays 4600 ZAR each month.

(c) Find the total number of monthly payments he will need to make to pay off the loan.

This strategy will result in Ruben's final payment being less than 4600 ZAR.

(d) Determine the amount of Ruben's final payment, to two decimal places.

(e) Hence, determine the total amount Ruben will save, to the nearest ZAR , by making the higher monthly payments.

Q30. Daisy invested 37000 Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

(a) Calculate the value of Daisy's investment after 2 years.

After m months, the amount of money in the fixed deposit account has appreciated to more than 50 000 AUD

(b) Find the minimum value of m where $m \in \mathbb{N}$.

Daisy is saving to purchase a new apartment. The price of the apartment is 200000 AUD.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

(c) Write down the amount of the loan.

The loan is for 10 years, compounded monthly, with equal monthly payments of 1700 AUD made by Daisy at the end of each month.

For this loan, find

(d) i) the amount of interest paid by Daisy.

(d.ii) the annual interest rate of the loan

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

(e) Find the amount of Daisy's final payment.

(f) Find how much money Daisy saved by making one final payment after 5 years.

1.8	
Use technology to solve: Systems of linear equations in up to 3 variables Polynomial equations	In examinations, no specific method of solution will be required. In examinations, there will always be a unique solution to a system of equations. Standard terminology, such as zeros or roots, should be taught.

Q31.

Solve:

$3x + 2y + 4z = -1$

$x + y + z = 0$

$10x + 7y + 4z = 6$

Q32.

Find the roots of the equation $x^3 - 4x^2 + 2x + 1 = 0$.

Q33.

3000 citizens attended a carnival. Let x be the number of adults attending the carnival and y be the number of children attending the carnival.

(a) Write down an equation in x and y.

[1]

The cost of an adult ticket and a child ticket were set to be USD 18 and USD 8 respectively. The total cost of tickets sold in the carnival was USD 36000.

(b) Write down another equation in x and y.

[1]

(c) Write down the values of x and y.

[2]

(d) Find the total cost for a group of 4 adults and 7 children.

[2]

Q34.

The total profit $\$y$ of selling plastic boxes can be modelled by the equation $y = ax^2 + bx + c$, where x is the number of plastic boxes ordered from a factory, and a, b and c are real numbers. The total profits of selling 100 plastic boxes, 200 plastic boxes and 400 plastic boxes are \$122, \$424 and \$1628 respectively.

(a) (i) Show that $10000a + 100b + c = 122$.

(ii) Show that $40000a + 200b + c = 424$.

(iii) Write down the third equation in a, b and c.

[3]

(b) Hence, find the values of a, b and c.

[4]

Q35.

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, $)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was \$7816.
- There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x, the number of child tickets sold be y, and the number of student tickets sold be z.

(a) Write down three equations that express the information given above.

(b) Find the number of each type of ticket sold.