MARKING SCHEME 3

1. 25M.1.SL.TZ1.8

(a)

Write $f(x)$ in the form $a(x-h^2)+k$, where $a, h, k \in \mathbb{Z}$.

[4]

Markscheme

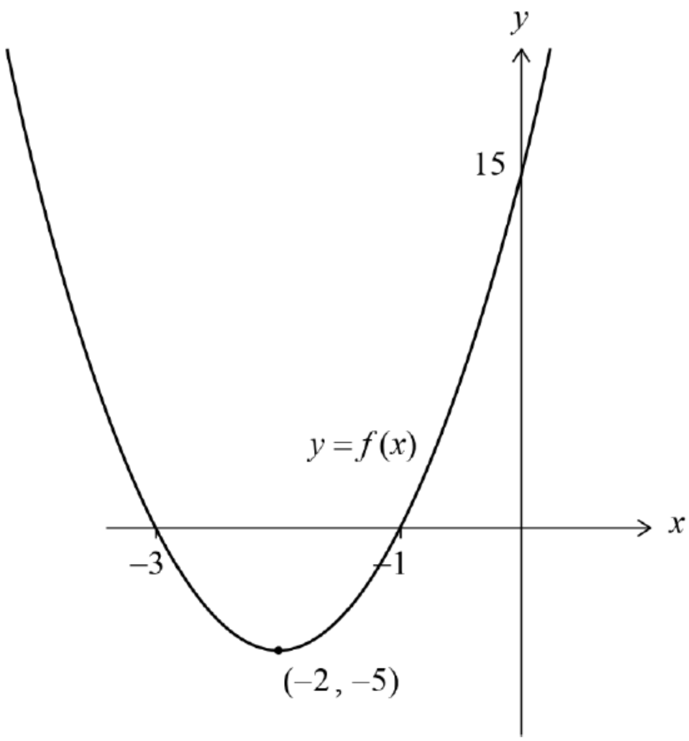
METHOD 1 $a=5$ ***(A1)*** attempt to use roots and symmetry to find h ***(M1)*** $h=\frac{(-1)+(-3)}{2}$ **OR** half the distance between the roots $\frac{(-1)-(-3)}{2}=1$ (may be seen on a diagram) $h=-2$ (accept $x=-2$) ***(A1)*** $f(x)=5(x-(-2))^2-5(=5(x+2)^2-5)$ ***A1*** $(a=5,\ h=-2,\ k=-5)$ **METHOD 2** $a=5$ ***(A1)*** attempt to expand $(x+1)(x+3)=x^2+4x+3$ **OR** $5(x+1)(x+3)=5x^2+20x+15$ **EITHER** uses their expansion to attempt to complete the square to the form ***(M1)*** $p(x+q)^2+r$, where q is half the coefficient of their x term $=(x+2)^2-2^2+3(=(x+2)^2-1)$ **OR** $5[(x+2)^2-2^2+3](=5(x+2)^2-5)$ ***(A1)*** **OR** uses their expansion to attempt to differentiate and sets equal to zero ***(M1)*** $\frac{dy}{dx}=2x+4=0$ **OR** $\frac{dy}{dx}=10x+20=0$ $h=-2$ (accept $x=-2$) ***(A1)*** **OR** uses their expansion to attempt to find axis of symmetry using $h=\frac{-b}{2a}$ ***(M1)*** $h=\frac{-4}{2}$ **OR** $h=\frac{-20}{10}$ $h=-2$ (accept $x=-2$) ***(A1)*** **THEN** $f(x)=5(x-(-2))^2-5(=5(x+2)^2-5)$ ***A1*** $(a=5,\ h=-2,\ k=-5)$
[4 marks]

(b)

Sketch the graph of $y=f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex.

[4]

Markscheme



M1A1A1A1
award ***M1*** for a roughly symmetric curve which is concave up award ***A1*** for x intercepts at -3 and -1 award ***A1*** for y intercept at 15 award ***A1*** for vertex at $(-2,-5)$
[4 marks]

(c)

Solve the inequality $f(x) \leq 40$.

[4]

Markscheme

$5(x+2)^2 - 5 \leq 40$ **OR** $5(x+1)(x+3) \leq 40$ **OR** $(x+1)(x+3) \leq 8$ leading to $(x+2)^2 \leq 9$ **OR** $5x^2 + 20x - 25 \leq 0$ **OR** $x^2 + 4x - 5 \leq 0$ ***(A1)***

valid attempt to find the critical values for their quadratic inequality ***(M1)*** $x + 2 = \pm 3$ **OR** $(x+5)(x-1) = 0$ $x = -5, x = 1$ ***(A1)*** $-5 \leq x \leq 1$ ***A1***

Note: Accept $(x \in)[-5,1]$ or equivalent.

[4 marks]

(d.i)

Write down an expression for $(f \circ g)(x)$.

[1]

Markscheme
$(f \circ g)(x) = 5(\ln x + 1)(In\, x + 3)$ **OR** $5(\ln x + 2)^2 - 5$ **OR** $5(\ln x)^2 + 20\ln x + 15$ ***A1***   ***[1 mark]***

(d.ii)

Solve the inequality $(f \circ g)(x) \leq 40$.

[2]

Markscheme
attempt to replace x with $\ln x$ using their solution to part (c) ***(M1)*** $-5 \leq \ln x \leq 1$ $e^{-5} \leq x \leq e$ ***A1*** **Note:** Accept $(x \in)[e^{-5}, e]$ or equivalent.   ***[2 marks]***

2. 25M.1.SL.TZ1.9

(a.i)

By considering triangle ABC, show that $R = r\cos\theta$.

[2]

Markscheme
recognition that $AB = 2r$ and $BC = 2R$ (seen anywhere) ***(M1)*** $\cos\theta = \frac{2R}{2r}$ **OR** $\frac{2r}{\sin 90°} = \frac{2r}{\sin(90° - \theta)}\left(= \frac{2R}{\cos\theta}\right)$ ***A1*** $R = r\cos\theta$ ***AG***   ***[2 marks]***

(a.ii)

Find an expression for h in terms of r and θ.

[2]

Markscheme
attempt to use Pythagoras, sine or the sine rule or cosine rule in triangle $A\hat{B}C$ or similar ***(M1)*** $\sin\theta = \frac{h}{2r}$ **OR** $h^2 + (2R)^2 = (2r)^2$ **OR** $\frac{2r}{\sin 90°} = \frac{h}{\sin\theta}$ **OR** $h^2 = (2R)^2 + (2r)^2 - 2 \times 2R \times 2r\cos(\theta)$ $h = 2r\sin\theta$ **OR** $h = \sqrt{4r^2 - 4r^2\cos^2\theta}$ ***A1*** ***[2 marks]***

(b)

Hence or otherwise, show that the total surface area, S cm^2, of the cylinder is given by $S = 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$.

[4]

Markscheme
area of one circle $= \pi R^2 = \pi(r\cos\theta)^2$ ***A1*** curved surface area $= 2\pi Rh = 2\pi(r\cos\theta)(2r\sin\theta)$ **OR** $= 2\pi(r\cos\theta)\sqrt{4r^2 - 4r^2\cos^2\theta}$ ***A1*** **Note:** these ***A1*** marks can be awarded independently. recognition that total surface area = area of two circles + curved surface area $= 2\pi R^2 + 2\pi Rh$ (seen anywhere) ***M1*** $= 2\pi r^2\cos^2\theta + 4\pi r^2\sin\theta\cos\theta$ **OR** $= 2\pi r^2\cos^2\theta + 2\pi r\cos\theta\sqrt{4r^2 - 4r^2\cos^2\theta}$ $= 2\pi r^2(1 - \sin^2\theta) + 4\pi r^2\sin\theta\cos\theta$ ***A1*** total surface area of the cylinder $= 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$ ***AG*** ***[4 marks]***

(c)

Show that $\tan\theta = 2$.

[4]

Markscheme
attempt to equate external surface area of the sphere to $2S$ $4\pi r^2 = 4\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$ or equivalent ***M1*** $2\sin\theta\cos\theta - \sin^2\theta = 0$ ***A1*** $\sin\theta \neq 0$ ***R1*** $2\cos\theta - \sin\theta = 0$ ***A1*** $\tan\theta = 2$ ***AG***   ***[4 marks]***

(d)

Find V, giving your answer in the form $p\pi r^3\sqrt{5}$, where $p \in \mathbb{Q}^+$.

[5]

Markscheme
attempt to find volume of the cylinder in terms of r and θ ***(M1)*** Volume of cylinder $= \pi R^2 h =$ $\pi(r\cos\theta)^2(2r\sin\theta)$ **OR** $\pi r^2\cos^2\theta\sqrt{4r^2 - 4r^2\cos^2\theta}$ **OR** $\pi(r\cos\theta)^2 \times 4r\cos\theta$ ***A1*** attempt to use right angled triangle to find $\cos\theta$ and $\sin\theta$ ***(M1)*** $\sqrt{1^2 + 2^2} = \sqrt{5},\ \cos\theta = \frac{1}{\sqrt{5}},\ \sin\theta = \frac{2}{\sqrt{5}}$ volume $= 2\pi r^3\left(\frac{1}{\sqrt{5}}\right)^2\left(\frac{2}{\sqrt{5}}\right)$ **OR** $4\pi r^3\left(\frac{1}{\sqrt{5}}\right)^3$ ***A1*** $= \frac{4}{25}\pi r^3\sqrt{5}$ ***A1*** $p = \frac{4}{25}$ ***[5 marks]***

3. 24N.1.SL.TZ1.8

(a)

Find the value of

[[N/A]]

(a.i)

$f(2)$;

[2]

Markscheme
substitution of $x = 2$ in part (i) or $x = \frac{1}{8}$ in part (ii) ***(M1)*** $\log_2(8 \times 2)$ OR $\log_2(16)$ $\log_2(16) = 4$ ***A1*** ***[2 marks]***

(a.ii)

$f\left(\frac{1}{8}\right)$.

[1]

Markscheme
$f\left(\frac{1}{8}\right) = 0$ ***A1*** ***[1 mark]***

(b)

Find an expression for $f^{-1}(x)$.

[4]

Markscheme
swap x and y ***(M1)*** $x = \log_2(8y)$ OR $x = 3 + \log_2 y$ attempt to write as exponential ***(M1)*** $2^x = 8y$ ***(A1)*** $(f^{-1}(x) =)\ \frac{2^x}{8}$ $(= 2^{x-3})$ (accept $y = \frac{2^x}{8}$ or $y = 2^{x-3}$) ***A1*** ***[4 marks]***

(c)

Hence, or otherwise, find $f^{-1}(0)$.

[1]

Markscheme
$\frac{1}{8}$ ***A1***   ***[1 mark]***

(d)

Describe these two transformations specifying the order in which they are to be applied.

[6]

Markscheme
METHOD 1 $f(4x^2) = \log_2(8 \times 4x^2)$ attempt to use addition rule for logs ***(M1)*** $\log_2 8 + \log_2 4 + \log_2 x^2$ OR $\log_2 32 + \log_2 x^2$ (or equivalent) ***(A1)*** attempt to use exponent property for logarithms ***(M1)*** $f(4x^2) = 5 + 2\log_2 x$ (or equivalent) ***A1*** the graph of g must be vertically stretched (dilated) by a scale factor of 2 and then vertically translated (shifted) 5 units upwards. ***A2***   **METHOD 2** $f(4x^2) = \log_2(8 \times 4x^2)$ attempt to write argument as a power ***(M1)*** $\log_2(32x^2) = \log_2\left(\left(\sqrt{32}x\right)^2\right)$ (or equivalent) ***(A1)*** attempt to use exponent property for logarithms ***(M1)*** $f(4x^2) = 2\log_2\left(\sqrt{32}x\right)$ (or equivalent) ***A1*** **EITHER** the graph of g must be vertically stretched (dilated) by a scale factor of 2 and stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$. ***A1*** **OR** the graph of g must be stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$ and vertically stretched (dilated) by a scale factor of 2. ***A1***   **Note:** In this method, the final mark is ***A1***, as the question specifically asks for a translation and a stretch.   ***[6 marks]***

4. 24N.1.SL.TZ1.9

(a)

Show that $S = 30\pi r^2 + 10\pi rh$.

[3]

Markscheme
outer curved surface area is $2\pi(4r)h$ AND inner curved surface area is $2\pi rh$ ***(A1)*** area of each base (top and bottom) is $\pi(4r)^2 - \pi r^2$ ***(A1)*** $S = 2[\pi(4r)^2 - \pi r^2] + 2\pi(4r)h + 2\pi rh$ ***A1*** $= 30\pi r^2 + 10\pi rh$ ***AG*** ***[3 marks]***

(b)

The total surface area of the hollow cylinder is 240π cm^2.

Show that $V = 360\pi r - 45\pi r^3$.

[6]

Markscheme
$30\pi r^2 + 10\pi rh = 240\pi$ attempt to solve their equation for h or rh in terms of r (must isolate h or rh) ***(M1)*** $h = \frac{240-30r^2}{10r}$ $\left(= \frac{24-3r^2}{r}\right)$ OR $rh = \frac{240-30r^2}{10}$ $(= 24 - 3r^2)$ (or equivalent) ***A1*** uses volume = large cylinder − small cylinder ***(M1)*** $V = \pi(4r)^2h - \pi r^2h$ $(= 16\pi r^2h - \pi r^2h = 15\pi r^2h)$ ***A1*** attempt to substitute in for h or rh ***(M1)*** $V = 15\pi r^2\left(\frac{24-3r^2}{r}\right)$ OR $V = 15\pi r\left(\frac{240-30r^2}{10}\right)$ $\left(= 15\pi r(24 - 3r^2)\right)$ OR $384\pi r - 48\pi r^3 - 24\pi r + 3\pi r^3$ ***A1*** $= 360\pi r - 45\pi r^3$ ***AG*** ***[6 marks]***

(c)

Find an expression for $\frac{dV}{dr}$.

[2]

Markscheme

$\frac{dV}{dr} = 360\pi - 135\pi r^2$ ***A1A1***
[2 marks]

(d)

Find the value of p.

[3]

Markscheme

METHOD 1 (working with r) recognition that (for a maximum) $\frac{dV}{dr} = 0$ ***M1*** $360\pi - 135\pi r^2 = 0$ $r^2 = \frac{360}{135}$ $\left(= \frac{8}{3}\right)$ $r = \sqrt{\frac{360}{135}}$ $\left(= \sqrt{\frac{8}{3}}\right)$ ***A1*** $p = 2$ OR $r = 2\sqrt{\frac{2}{3}}$ ***A1*** **METHOD 2** (working with $p\sqrt{\frac{2}{3}}$) recognition that (for a maximum) $\frac{dV}{dr} = 0$ ***M1*** $360\pi - 135\pi\left(p\sqrt{\frac{2}{3}}\right)^2 = 0$ $360 - 90p^2 = 0$ $p^2 = 4$ ***A1*** $p = 2$ OR $r = 2\sqrt{\frac{2}{3}}$ ***A1***
[3 marks]

(e)

Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$.

[3]

Markscheme

attempt to substitute their value of r into $V = 360\pi r - 45\pi r^3$ ***M1*** $V = 360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times \left(2\sqrt{\frac{2}{3}}\right)^3 = 360\pi \times 2\sqrt{\frac{2}{3}} - 45\pi \times 8 \times \frac{2}{3} \times \sqrt{\frac{2}{3}}$ $\left(= 720\pi\sqrt{\frac{2}{3}} - 240\pi\sqrt{\frac{2}{3}}\right)$ ***(A1)*** $= 480\pi\sqrt{\frac{2}{3}}$ ***A1*** $(q = 480)$
[3 marks]

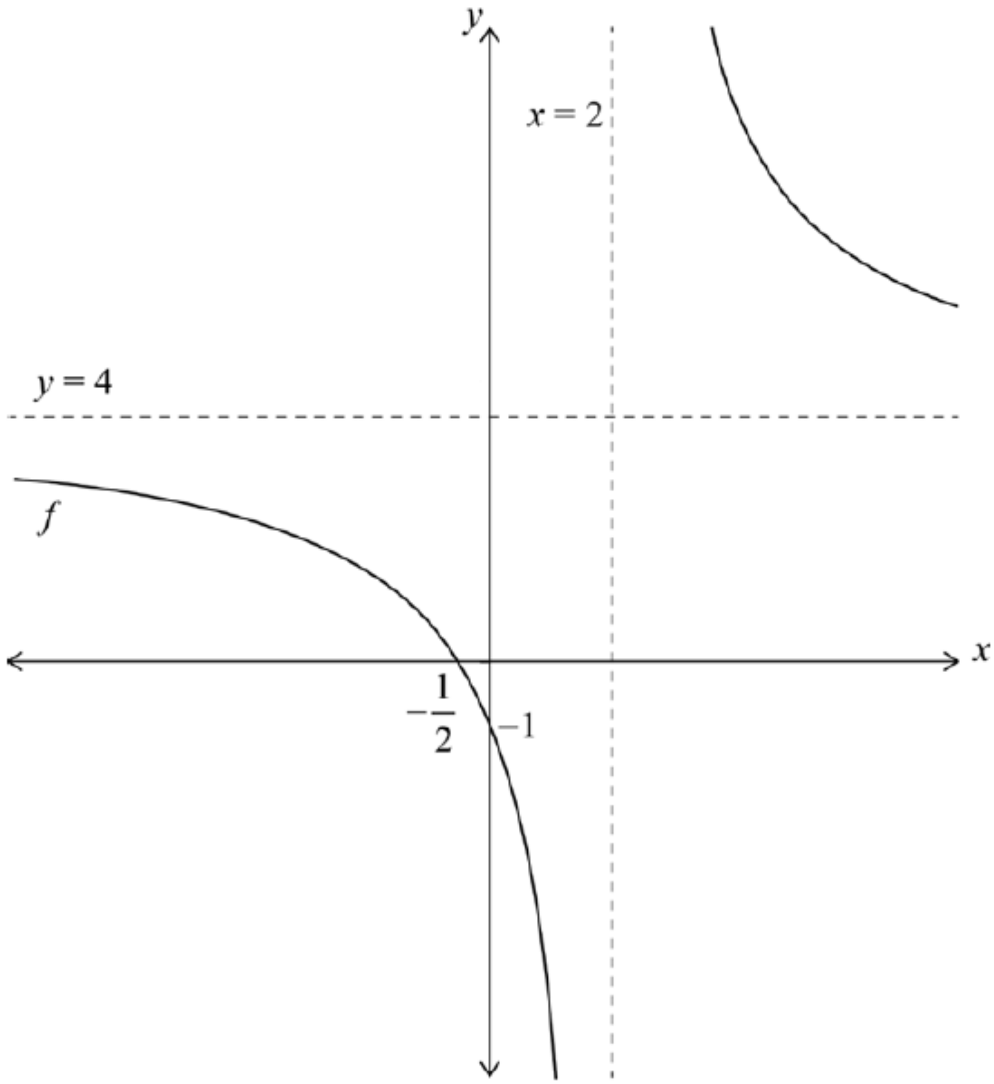
5. 24M.1.SL.TZ1.8

(a)

Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations.

[5]

Markscheme



vertical asymptote $x = 2$ sketched and labelled with correct equation ***A1*** horizontal asymptote $y = 4$ sketched and labelled with

correct equation ***A1*** For an approximate rational function shape: labelled intercepts $-\frac{1}{2}$ on x-axis, -1 on y-axis ***A1A1*** two branches in correct opposite quadrants with correct asymptotic behaviour ***A1*** **Note:** These marks may be awarded independently.
[5 marks]

(b)

Write down the range of f.

[1]

Markscheme
$y \neq 4$ (or equivalent) ***A1***   ***[1 mark]***

(c)

Show that $p = \frac{9}{2}$.

[1]

Markscheme
$2 + \frac{5}{2}$ OR $\left(-\frac{1}{2}\right) + 2 \times \frac{5}{2}$ OR $\frac{-\frac{1}{2}+p}{2} = 2$ OR $-4 = -p + \frac{1}{2}$ ***A1*** $p = \frac{9}{2}$ ***AG***   ***[1 mark]***

(d)

Find the value of b and the value of c.

[3]

Markscheme

METHOD 1 attempt to substitute both roots to form a quadratic ***(M1)*** **EITHER** $\left(x+\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$ OR $x^2-\left(-\frac{1}{2}+\frac{9}{2}\right)x+\left(-\frac{1}{2}\times\frac{9}{2}\right)=x^2-4x-\frac{9}{4}$ ***A1A1*** $\left(b=-4, c=-\frac{9}{4}\right)$ **Note:** Award ***A1*** for each correct value. They may be embedded or stated explicitly. **OR** $(2x+1)(2x-9)=4\left(x^2-4x-\frac{9}{4}\right)$ $b=-4, c=-\frac{9}{4}$ ***A1A1*** **Note:** Award ***A1*** for each correct value. They must be stated explicitly. **METHOD 2** $-\frac{b}{2}=2$ OR $4+b=0\Rightarrow b=-4$ ***A1*** attempt to form a valid equation to find c using their b ***(M1)*** $\left(-\frac{1}{2}\right)^2+-4\left(-\frac{1}{2}\right)+c=0$ OR $\left(\frac{9}{2}\right)^2+-4\left(\frac{9}{2}\right)+c=0$ $c=-\frac{9}{4}$ ***A1*** **METHOD 3** attempt to form two valid equations in b and c ***(M1)*** $\left(-\frac{1}{2}\right)^2+b\left(-\frac{1}{2}\right)+c=0$, $\left(\frac{9}{2}\right)^2+b\left(\frac{9}{2}\right)+c=0$ $b=-4, c=-\frac{9}{4}$ ***A1A1*** **METHOD 4** attempt to write $g(x)$ in the form $(x-h)^2+k$ and substitute for x, h and $g(x)$ ***(M1)*** $\left(-\frac{1}{2}-2\right)^2+k=0\Rightarrow k=-\frac{25}{4}$ $(x-2)^2-\frac{25}{4}=x^2-4x-\frac{9}{4}$ ***A1A1*** $\left(b=-4, c=-\frac{9}{4}\right)$ **Note:** Award ***A1*** for each correct value. They may be embedded or stated explicitly.
[3 marks]

(e)

Find the y-coordinate of the vertex of the graph of $y=g(x)$.

[2]

Markscheme

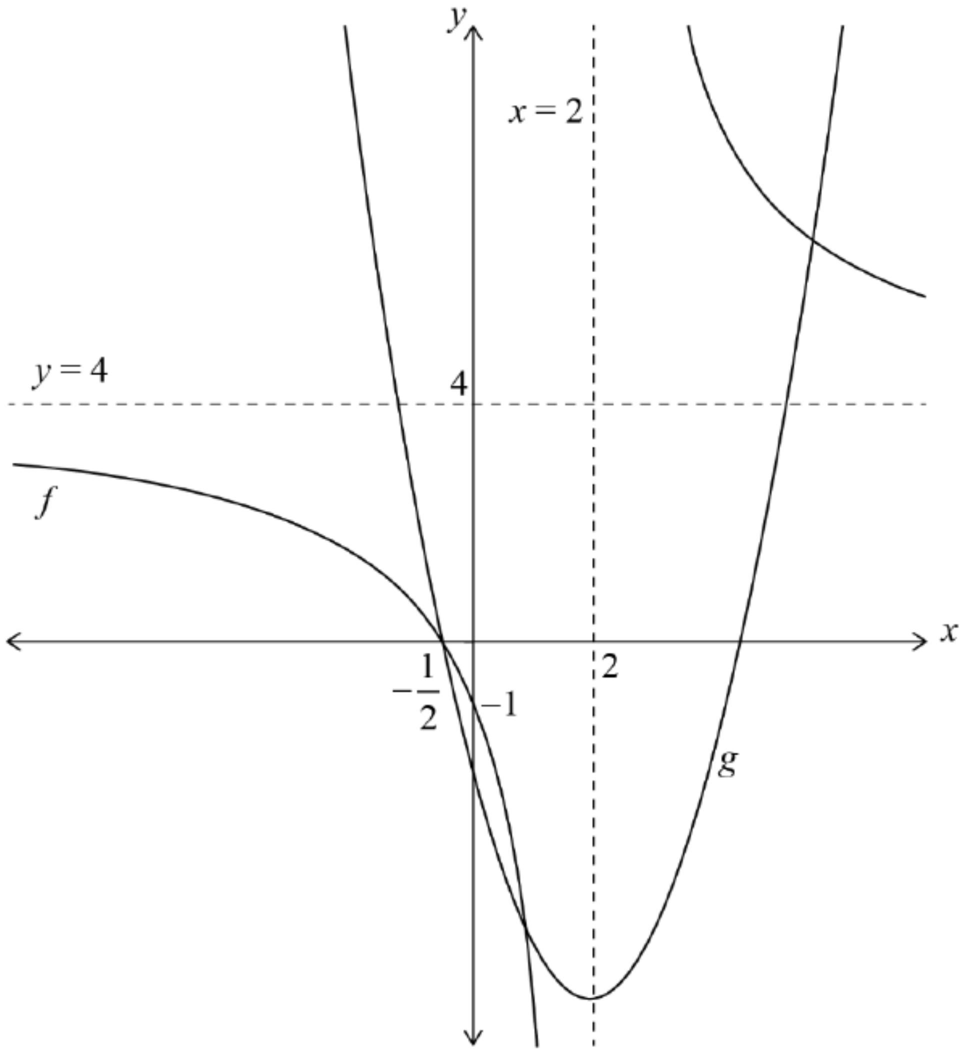
attempt to substitute $x=2$ into their $g(x)$ OR complete the square on their $g(x)$ (may be seen in part (d)) ***(M1)*** $y=-\frac{25}{4}$ ***A1***
[2 marks]

(f)

Find the number of solutions of the equation $f(x)=g(x)$.

[2]

Markscheme



both graphs sketched on same axes and identifying points of intersection ***(M1)*** 3 solutions ***A1*** **Note:** Exception to ***FT***: If the candidate's graph in part (a) is incorrect, the ***M1*** may be awarded for a sketch of their graph from part (a) and $g(x)$. Do not award the final ***A1*** in this case.

[2 marks]

6. 24M.1.SL.TZ1.9

(a)

Find the value of p and the value of q.

[2]

Markscheme
evidence of understanding that there are now $3R$ and $2B$ ***(M1)*** $p = \frac{3}{5},\ q = \frac{2}{5}$ ***A1***   ***[2 marks]***

(b)

Show that the probability that Francine selects two buttons of the same colour is $\frac{7}{10}$.

[2]

Markscheme
attempt to add two products ***(M1)*** P(same) = P(RR or BB) $= \frac{3}{4} \times \frac{4}{5} + \frac{1}{4} \times \frac{2}{5}$ ***A1*** $= \frac{14}{20} = \frac{7}{10}$ ***AG***   ***[2 marks]***

(c)

Given that Francine selects two buttons of the same colour, find the probability that she selects two red buttons.

[3]

Markscheme
attempt to use conditional probability formula in context ***(M1)*** $P(RR\|same) = \frac{P(RR)}{P(same)}$ **Note:** Award ***M0*** if candidate only writes $P(A\|B)$ formula and nothing else. $= \frac{\left(\frac{12}{20}\right)}{\left(\frac{14}{20}\right)}$ ***(A1)*** $= \frac{12}{14}\left(= \frac{6}{7}\right)$ ***A1***   ***[3 marks]***

(d)

Find the value of a and the value of b.

[2]

Markscheme
$a = \frac{6}{20}\left(= \frac{3}{10}\right)$, $b = \frac{12}{20}\left(= \frac{6}{10}\right)$ ***A1A1*** ***[2 marks]***

(e)

Hence, find the expected number of red buttons selected by Francine.

[2]

Markscheme
attempt to use the formula for $E(X)$ ***(M1)*** $E(X) = 0 \times \frac{1}{10} + 1 \times \frac{6}{20} + 2 \times \frac{12}{20} = \frac{30}{20}\left(= \frac{3}{2}\right)$ ***A1*** ***[2 marks]***

(f)

Given that the first two buttons she selects are red, write down the probability that the next button she selects is blue.

[1]

Markscheme
$\frac{1}{6}$ ***A1*** ***[1 mark]***

(g)

Find the value of n.

[5]

Markscheme

METHOD 1 $P(n-1\ reds)=\frac{3}{4}\times\frac{4}{5}\times\ldots\times\frac{n+1}{n+2}\left(=\frac{3}{n+2}\right)$ ***(A1)*** $P(next\ one\ blue)=\frac{1}{n+3}$ ***(A1)*** $P(n-1\ reds\ then\ 1\ blue)=P(n-1\ reds)\times P(next\ one\ blue)$ ***(M1)*** $\frac{3}{n+2}\times\frac{1}{n+3}=\frac{3}{56}$ ***(A1)*** $(n+2)(n+3)=56$ $n=5$ ***A1*** **Note:** If no working shown, award ***M1A0A0A0A1*** for $n=5$. **METHOD 2** Let X be the number of selections in total made when first blue picked attempt to establish pattern for $X=1,2,3,\ldots$ with at least 3 cases ***(M1)*** $P(X=1)=\frac{1}{4}$ and $P(second\ pick)=\frac{3}{4}\times\frac{1}{5}$ ***(A1)*** $P(X=3)\left(=\frac{3}{4}\times\frac{4}{5}\times\frac{1}{6}\right)=\frac{3}{5}\times\frac{1}{6}$ ***(A1)*** $P(X=5)=\frac{3}{7}\times\frac{1}{8}\left(=\frac{3}{56}\right)$ ***(A1)*** so $n=5$ ***A1*** **METHOD 3** $P(next\ one\ blue)=\frac{1}{n+3}$ ***(A1)*** recognising $P(n-1\ R\ then\ 1B)=P(n-1\ R)\times P(next\ one\ B)$ OR $\frac{3}{4}\times\frac{4}{5}\times\ldots\times\frac{1}{n+3}$ ***(M1)*** $\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}\times\frac{1}{8}\left(=\frac{3}{56}\right)$ ***(A1)(A1)*** **Note:** Award ***A1*** for $\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}$ (seen anywhere) and ***A1*** for $\times\frac{1}{8}$. so $n=5$ ***A1***

[5 marks]

7. 23N.1.SL.TZ1.8

[[N/A]]

(a)

State the equation of the vertical asymptote to the graph of $y=g(x)$.

[1]

Markscheme

$x=0$ ***A1***

[1 mark]

(b.i)

Show that, at the points of intersection, $x^2-2dx+7d=0$.

[4]

Markscheme
setting $\ln(2x-7) = 2\ln x - \ln d$ ***M1*** attempt to use power rule ***(M1)*** $2\ln x = \ln x^2$ (seen anywhere) attempt to use product/quotient rule for logs ***(M1)*** $\ln(2x-7) = \ln\frac{x^2}{d}$ OR $\ln\frac{x^2}{2x-7} = \ln d$ OR $\ln(2x-7)d = \ln x^2$ $\frac{x^2}{d} = 2x-7$ OR $\frac{x^2}{2x-7} = d$ OR $(2x-7) = x^2$ ***A1*** $x^2 - 2dx + 7d = 0$ ***AG*** ***[4 marks]***

(b.ii)

Hence, show that $d^2 - 7d > 0$.

[3]

Markscheme
discriminant = $(-2d)^2 - 4 \times 7d$ ***(A1)*** recognizing discriminant > 0 ***(M1)*** $(2d)^2 - 4 \times 7d > 0$ OR $4d^2 - 28d > 0$ ***A1*** $d^2 - 7d > 0$ ***AG*** ***[3 marks]***

(b.iii)

Find the range of possible values of *d*.

[2]

Markscheme
setting $d(d-7) > 0$ OR $d(d-7) = 0$ OR sketch graph OR sign test OR $d^2 > 7d$ ***(M1)*** $d < 0$ or $d > 7$, but $d \in \mathbb{R}^+$ $d > 7$ (or $]7, \infty[$) ***A1*** ***[2 marks]***

(c)

In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a,\ b \in \mathbb{Z}^+$.

[5]

Markscheme

$x^2 - 20x + 70\ (= 0)$ ***A1*** attempting to solve their 3 term quadratic equation ***(M1)*** $((x-10)^2 - 30 = 0)$ or $\left((x =)\frac{20 \pm \sqrt{(-20)^2 - 4 \times 1 \times 70}}{2}\right)$ $x = 10 - \sqrt{30}(= p)$ or $x = 10 + \sqrt{30}(= q)$ ***(A1)*** subtracting their values of x ***(M1)***
distance $= 2\sqrt{30}\ (or\ \sqrt{120})$ **A1** $(a = 2,\ b = 30)\ (or\ a = 1,\ b = 120)$

[5 marks]

8. 23N.1.SL.TZ1.9

(a)

Show that $f'(x) = \frac{12(2-5x^2)}{(x^2+2)^4}$.

[4]

Markscheme

attempt to use either the quotient or product rule ***(M1)***
$\frac{12(x^2+2)^3 - 12x \times 3 \times 2x(x^2+2)^2}{(x^2+2)^6}$ OR $12(x^2+2)^{-3} + 12x \times (-3) \times 2x(x^2+2)^{-4}$ ***A1A1*** **Note**: Award ***A1*** for correctly applying chain rule to $(x^2+2)^3$ and ***A1*** for everything else correct. =
$\frac{12(x^2+2-6x^2)}{(x^2+2)^4}$ OR $\frac{12(x^2+2)^2(x^2+2-6x^2)}{(x^2+2)^6}$ OR $\frac{24-60x^2}{(x^2+2)^4}$ OR $\frac{12x^2+24-72x^2}{(x^2+2)^4}$
A1
$= \frac{12(2-5x^2)}{(x^2+2)^4}$ ***AG***

[4 marks]

(b)

Find $\int f(x)\,dx$.

[4]

Markscheme
EITHER attempts to integrate by substitution using $u = x^2 + 2$ or $u = x^2$ ***(M1)*** $u = x^2 + 2 \Rightarrow \frac{du}{dx} = 2x$ OR $u = x^2 \Rightarrow \frac{du}{dx} = 2x$ **Note**: If candidate simply states $u = x^2 + 2$ or $u = x^2$, but but does not attempt to substitute into their integral, do not award the ***(M1)***. $\int \frac{12x}{(x^2+2)^3}\, dx = \int \frac{6}{u^3} du$ OR $\int \frac{12x}{(x^2+2)^3}\, dx = \int \frac{6}{(u+2)^3}\, du$ ***(A1)*** $= 3u^{-2}(+c)$ OR $-3(u+1)^{-2}(+c)$ ***(A1)*** **OR** attempts to apply integration by inspection ***(M1)*** $6\int \frac{2x}{(x^2+2)^3}\, dx = 6x\left(-\frac{1}{2}\right)(x^2 + 2)^{-2}\ (+c)$ ***(A1)(A1)*** **Note:** Award ***A1*** for correct power of $(x^2 + 2)$ and ***A1*** for $-\frac{1}{2}$. **THEN** $-3(x^2 + 2)^{-2} + c$ OR $-\frac{3}{(x^2+2^2)} + c$ (final answer must include $+c$) ***A1*** ***[4 marks]***

(c)

Find the two possible expressions for $g(x)$.

[5]

Markscheme
recognizing $g'(x) = f'(x) \Rightarrow g(x) = f(x) + k$ (may be seen in diagram/drawing) ***A1*** area of R is given by subtracting functions f and g in integral(s) ***(M1)*** $\pm\int_0^3 k\ dx$ OR $=\int_0^3 \lvert g - f\rvert\ dx$ OR $\int_0^3 f(x) + k - f(x)\, dx$ OR $\int_0^3 f(x)\, dx - \int_0^3 g(x)\, dx =$ $\pm[kx]_0^3$ OR $\left[-\frac{2}{(x^2+1)^2} + kx\right]_0^3 - \left[-\frac{2}{(x^2+1)^2}\right]_0^3$ OR $\left[-\frac{2}{(x^2+1)^2}\right]_0^3 - \left[-\frac{2}{(x^2+1)^2} + kx\right]_0^3$ ***(A1)*** $\pm 3k = \frac{21}{2}$ ***(A1)*** $k = \pm\frac{21}{6}\left(= \pm\frac{7}{2} = \pm 3.5\right)$ $g(x) = \frac{12x}{(x^2+2)^3} - \frac{7}{2}$ AND $g(x) = \frac{12x}{(x^2+2)^3} + \frac{7}{2}$ (accept $f(x) + \frac{7}{2}$ AND $f(x) - \frac{7}{2}$) ***A1*** ***[5 marks]***

9. 23M.1.SL.TZ1.8

[[N/A]]

(a.i)

Find the sum of the first five terms.

[2]

Markscheme
recognition that $n = 5$ ***(M1)*** $S_5 = 45$ ***A1 [2 marks]***

(a.ii)

Given that $S_6 = 60$, find u_6.

[2]

Markscheme
METHOD 1 recognition that $S_5 + u_6 = S_6$ ***(M1)*** $u_6 =$ 15 ***A1*** **METHOD 2** recognition that $60 = \frac{6}{2}(S_1 + u_6)$ ***(M1)*** $60 =$ $3(5 + u_6)$ $u_6 = 15$ ***A1*** **METHOD 3** substituting their u_1 and d values into $u_1 + (n-1)d$ ***(M1)*** $u_6 = 15$ ***A1***   ***[2 marks]***

(b)

Find u_1.

[2]

Markscheme
recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their u_6 into S_6 ***(M1)*** OR equations for S_5 and S_6 in terms of u_1 and d $1 +$ 4 OR $60 = \frac{6}{2}(U_1 + 15)$ $u_1 = 5$ ***A1 [2 marks]***

(c)

Hence or otherwise, write an expression for u_n in terms of n.

[3]

Markscheme
EITHER valid attempt to find d (may be seen in (a) or (b)) ***(M1)*** $d = 2$ ***(A1)*** **OR** valid attempt to find $S_n - S_{n-1}$ ***(M1)*** $n^2 + 4n - (n^2 - 2n + 1 + 4n - 4)$ ***(A1)*** **OR** equating $n^2 + 4n = \frac{n}{2}(5 + u_n)$ ***(M1)*** $2n + 8 = 5 + u_n$ (or equivalent) ***(A1)*** **THEN** $u_n = 5 + 2(n-1)$ OR $u_n = 2n + 3$ ***A1***   ***[3 marks]***

(d)

Find the possible values of the common ratio, r.

[3]

Markscheme
recognition that $v_2 r^2 = v_4$ OR $(v_3)^2 = v_2 \times v_4$ ***(M1)*** $r^2 = 3$ OR $v_3 = (\pm)5\sqrt{3}$ ***(A1)*** $r = \pm\sqrt{3}$ ***A1*** **Note:** If no working shown, award ***M1A1A0*** for $\sqrt{3}$. ***[3 marks]***

(e)

Given that $v_{99} < 0$, find v_5.

[2]

Markscheme
recognition that r is negative ***(M1)*** $v_5 = -15\sqrt{3}$ $\left(= -\frac{45}{\sqrt{3}}\right)$ ***A1*** ***[2 marks]***

10. 23M.1.SL.TZ1.9

[[N/A]]

(a)

Find the displacement of the object from the origin at $t = 1$.

[5]

Markscheme
attempt to integrate v (integration of at least one term) ***(M1)*** $(s(t) =) -\frac{1}{4}t^4 + \frac{7}{6}t^3 - t^2 + 6t\ (+C)$ ***A2*** **Note:** Award ***A1*** for at least two correct terms. substitution of $t = 1$ into their integrated expression ***(M1)*** displacement $= 5\frac{11}{12}\left(= \frac{71}{12}\right)\ (m)$ ***A1*** ***[5 marks]***

(b)

Find an expression for the acceleration of the object.

[2]

Markscheme
attempt to differentiate v (differentiation of at least one term) ***(M1)*** $a(t) = -3t^2 + 7t - 2$ ***A1*** ***[2 marks]***

(c)

Hence, find the greatest speed reached by the object before it comes to rest.

[5]

Markscheme
setting their $v'(t) = 0$ ***(M1)*** $-3t^2 + 7t - 2 = 0$ valid attempt to solve quadratic ***(M1)*** $(3t-1)(t-2) = 0$ OR $\frac{-7 \pm \sqrt{49-4(-3)(-2)}}{-6}$ $t = \frac{1}{3}, 2$ ($t = \frac{1}{3}$ may be omitted) ***(A1)*** substitute their largest positive t-value into $v(t)$ ***(M1)*** greatest speed is $8\ (ms^{-1})$ ***A1*** ***[5 marks]***

(d)

Find the greatest speed reached by the object for $0 \leq t \leq 4$.

[2]

Markscheme
attempt to check other boundary value at $t = 4$ ***(M1)*** $v(4) = -64 + 56 - 8 + 6\ (= -10)$ greatest speed is $10\, ms^{-1}$ ***A1 [2 marks]***

(e)

Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression.

[3]

Markscheme
identifying correct intervals where speed increases (may be seen in integral) ***(A1)(A1)*** $t = \frac{1}{3}$ to $t = 2$ and $t = k$ to $t = 4$ $\int_{\frac{1}{3}}^{2} v\,(t)dt + \int_{k}^{4} \lvert v(t) \rvert\, dt$ OR $\int_{\frac{1}{3}}^{2} v\, dt + \left\lvert \int_{k}^{4} vd\, t \right\rvert$ OR $\int_{\frac{1}{3}}^{2} v\,(t)dt - \int_{k}^{4} v\,(t)dt$ ***A1*** **Note:** Condone missing dt. ***[3 marks]***

11. 23M.1.SL.TZ2.8

[[N/A]]

(a)

Find the value of

[[N/A]]

(a.i)

$\sin \theta$;

[2]

Markscheme
attempt to use Pythagoras ***(M1)*** $\sin^2\theta + \left(\frac{2}{3}\right)^2 = 1$ OR $x^2 + 2^2 = 3^2$ OR right triangle with side 2 and hypotenuse 3 $\sin\theta = \frac{\sqrt{5}}{3}$ ***A1 [2 marks]***

(a.ii)

$\sin 2\theta$.

[2]

Markscheme
attempt to substitute into double-angle identity using their value of $\sin\theta$ ***(M1)*** $\sin 2\theta = 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$ $\sin 2\theta = \frac{4\sqrt{5}}{9}$ ***A1 [2 marks]***

(b)

Show that $b = \frac{3a}{4}$.

[2]

Markscheme
METHOD 1 (using values from part (a)) $\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$ attempt to use sine rule with their values from part (a) ***(M1)*** $\frac{b}{\left(\frac{\sqrt{5}}{3}\right)} = \frac{a}{\left(\frac{4\sqrt{5}}{9}\right)}$ OR $\frac{\left(\frac{\sqrt{5}}{3}\right)}{b} = \frac{\left(\frac{4\sqrt{5}}{9}\right)}{a}$ correct working that leads to ***AG*** ***A1*** $\frac{\sqrt{5}}{3}a = \frac{4\sqrt{5}}{9}b$ OR $\frac{3b}{\sqrt{5}} = \frac{9a}{4\sqrt{5}}$ OR $\frac{a}{3} = \frac{4b}{9}$ (or equivalent) $b = \frac{3a}{4}$ ***AG*** **METHOD 2 (double-angle identity)** $\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$ using double-angle identity ***(A1)*** $\frac{b}{\sin\theta} = \frac{a}{2\sin\theta\cos\theta}$ OR $b = \frac{a\sin\theta}{2\sin\theta\cos\theta}$ OR $b = \frac{a}{2\cos\theta}$ correct working (involving substituting $\cos\theta = \frac{2}{3}$)

that leads to ***AG*** ***A1*** $b = \frac{a \sin\theta}{2\sin\theta\left(\frac{2}{3}\right)}$ OR $b = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)}$ OR $b = \frac{a}{2\left(\frac{2}{3}\right)}$ (or equivalent) $b = \frac{3a}{4}$ ***AG [2 marks]***

(c)

Find the value of $\sin C\hat{A}D$.

[3]

Markscheme

METHOD 1 (using supplementary angles) recognizing $C\hat{A}D$ and $B\hat{A}C$ are supplementary ***(M1)*** recognizing supplementary angles have the same sine value ***(A1)*** $\sin C\hat{A}D = \sin 2\theta$ $\sin C\hat{A}D = \frac{4\sqrt{5}}{9}$ ***A1*** **METHOD 2 (using sine rule)** recognizing $CD = a$ ***(M1)*** $\frac{a}{\sin C\hat{A}D} = \frac{b}{\sin\theta}$ correct substitution of $\sin\theta\,\frac{\sqrt{5}}{3}$ and $b = \frac{3a}{4}$ into sine rule ***(A1)*** $\frac{a}{\sin C\hat{A}D} = \frac{\left(\frac{3a}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)}$ **OR** $\sin C\hat{A}D = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{3a}{4}\right)}$ (or equivalent)

$\sin C\hat{A}D = \frac{4\sqrt{5}}{9}$ ***A1 [3 marks]***

(d)

Find the area of triangle DAC, in terms of a.

[5]

Markscheme

METHOD 1 (using $C\hat{A}D$ in area formula) recognizing $D\hat{C}A = \theta$ ***(A1)*** recognizing $AD = b \left(= \frac{3a}{4}\right)$ ***(A1)*** correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin C\hat{A}D$) ***(M1)*** $area = \frac{1}{2}(b)(b)\left(\frac{4\sqrt{5}}{9}\right)$ **OR** $area = \frac{1}{2}(b)(b)\sin 2\theta$ **OR** $area = \frac{1}{2}(b)(b)\sin C\hat{A}D$ correct substitution in terms of a ***(A1)*** $area = \frac{1}{2}\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\left(\frac{4\sqrt{5}}{9}\right)$ $area = \frac{\sqrt{5}a^2}{8}$ ***A1*** **METHOD 2 (using $A\hat{C}D$ or $A\widehat{D}C$ in area formula)** recognizing $CD = a$ ***(A1)***

recognizing $AD = b\left(=\frac{3a}{4}\right)$ and/or $D\hat{C}A = \theta$ ***(A1)*** correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin A\hat{D}C$ or $\sin A\hat{C}D$) ***(M1)*** $area = \frac{1}{2}(a)(b)\left(\frac{\sqrt{5}}{3}\right)$ OR $area = \frac{1}{2}(a)(b)\sin\theta$ OR $area = \frac{1}{2}(a)(b)\sin A\hat{D}C$ OR $area = \frac{1}{2}(a)(b)\sin A\hat{C}D$ correct substitution in terms of a ***(A1)***
$area = \frac{1}{2}(a)\left(\frac{3a}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$ $area = \frac{\sqrt{5}a^2}{8}$ ***A1 [5 marks]***

12. 23M.1.SL.TZ2.9

(a)

For point Q, show that $y = \sqrt{9 - x^2}$.

[1]

Markscheme

$y^2 = 9 - x^2$ OR $y = \pm\sqrt{9 - x^2}$ ***A1*** (since $y > 0$) $\Rightarrow y = \sqrt{9 - x^2}$ ***AG [1 mark]***

(b)

Hence, find an expression for A, the area of triangle PQR, in terms of x.

[3]

Markscheme

$b = 2y\left(= 2\sqrt{9 - x^2}\right)$ or $h = x + 3$ ***(A1)*** attempts to substitute their base expression and height expression into $A = \frac{1}{2}bh$ ***(M1)*** $A = \sqrt{9 - x^2}(x + 3)$ (or equivalent) $\left(= \frac{2(x+3)\sqrt{9-x^2}}{2} = x\sqrt{9 - x^2} + 3\sqrt{9 - x^2}\right)$ ***A1 [3 marks]***

(c)

Show that $\frac{dA}{dx}=\frac{9-3x-2x^2}{\sqrt{9-x^2}}$.

[4]

Markscheme

METHOD 1 attempts to use the product rule to find $\frac{dA}{dx}$ ***(M1)*** attempts to use the chain rule to find $\frac{d}{dx}\sqrt{9-x^2}$ ***(M1)*** $\left(\frac{dA}{dx}=\right)\sqrt{9-x^2}+(3+x)\left(\frac{1}{2}\right)(9-x^2)^{-\frac{1}{2}}(-2x)\left(=\sqrt{9-x^2}-\frac{x^2+3x}{\sqrt{9-x^2}}\right)$ ***A1*** $\left(\frac{dA}{dx}=\right)\frac{9-x^2}{\sqrt{9-x^2}}-\frac{x^2+3x}{\sqrt{9-x^2}}\left(=\frac{9-x^2-(x^2+3x)}{\sqrt{9-x^2}}\right)$ ***A1*** $\frac{dA}{dx}=\frac{9-3x-2x^2}{\sqrt{9-x^2}}$ ***AG*** **METHOD 2** $\frac{dA}{dx}=\frac{dA}{dy}\times\frac{dy}{dx}$ attempts to find $\frac{dA}{dy}$ where $A=y(x+3)$ and $\frac{dy}{dx}$ where $y^2=9-x^2$ ***(M1)*** $\frac{dA}{dy}=y\frac{dx}{dy}+x+3$ and $\frac{dy}{dx}=-\frac{x}{y}$ (or equivalent) ***A1*** substitutes their $\frac{dA}{dy}$ and their $\frac{dy}{dx}$ into $\frac{dA}{dx}=\frac{dA}{dy}\times\frac{dy}{dx}$ ***(M1)*** $\frac{dA}{dx}=\left(y\left(-\frac{y}{x}\right)+x+3\right)\left(-\frac{x}{y}\right)$ (or equivalent) $=\frac{9-x^2-x^2-3x}{\sqrt{9-x^2}}$ (or equivalent) ***A1*** $\frac{dA}{dx}=\frac{9-3x-2x^2}{\sqrt{9-x^2}}$ ***AG*** ***[4 marks]***

(d)

Hence or otherwise, find the y-coordinate of R such that A is a maximum.

[6]

Markscheme

$\frac{dA}{dx}=0\left(\frac{9-3x-2x^2}{\sqrt{9-x^2}}=0\right)$ ***(M1)*** attempts to solve $9-3x-2x^2=0$ (or equivalent) ***(M1)*** $-(2x-3)(x+3)(=0)$ OR $x=\frac{3\pm\sqrt{(-3)^2-4(-2)(9)}}{2(-2)}$ (or equivalent) ***(A1)*** $x=\frac{3}{2}$ ***A1*** **Note:** Award the above ***A1*** if $x=-3$ is also given. substitutes their value of x into either $y=\sqrt{9-x^2}$ or $y=-\sqrt{9-x^2}$ **Note:** Do not award the above ***(M1)*** if $x\leq 0$. ***(M1)*** $y=-\sqrt{9-\left(\frac{3}{2}\right)^2}=-\frac{\sqrt{27}}{2}\left(=-\frac{3\sqrt{3}}{2},=-\sqrt{\frac{27}{4}},=-\sqrt{6.75}\right)$ ***A1*** ***[6 marks]***

13. 22M.1.SL.TZ1.8

(a.i)

Show that $p = \pm\frac{1}{\sqrt{3}}$.

[2]

Markscheme

EITHER
attempt to use a ratio from consecutive terms ***M1***
$\frac{p\ln x}{\ln x} = \frac{\frac{1}{3}\ln x}{p\ln x}$ OR $\frac{1}{3}\ln x = (\ln x)r^2$ OR $p\ln x = \ln x\left(\frac{1}{3p}\right)$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}}$ ... and consider the powers of x in geometric sequence
Award ***M1*** for $\frac{p}{1} = \frac{\frac{1}{3}}{p}$.

OR
$r = p$ and $r^2 = \frac{1}{3}$ ***M1***

THEN
$p^2 = \frac{1}{3}$ OR $r = \pm\frac{1}{\sqrt{3}}$ ***A1***
$p = \pm\frac{1}{\sqrt{3}}$ ***AG***

Note: Award ***M0A0*** for $r^2 = \frac{1}{3}$ or $p^2 = \frac{1}{3}$ with no other working seen.

[2 marks]

(a.ii)

Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x.

[3]

Markscheme

$\frac{\ln x}{1-\frac{1}{\sqrt{3}}}$ $(= 3 + \sqrt{3})$ ***(A1)***
$\ln x = 3 - \frac{3}{\sqrt{3}} + \sqrt{3} - \frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x = 3 - \sqrt{3} + \sqrt{3} - 1$ $(\Rightarrow \ln x = 2)$ ***A1***
$x = e^2$ ***A1***

[3 marks]

(b.i)

Show that $p = \frac{2}{3}$.

[3]

Markscheme

METHOD 1

attempt to find a difference from consecutive terms or from u_2 ***M1***

correct equation ***A1***

$p \ln x - \ln x = \frac{1}{3}\ln x - p \ln x$ OR $\frac{1}{3}\ln x = \ln x + 2(p \ln x - \ln x)$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \cdots$ and consider the powers of x in arithmetic sequence.

Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$

$2p \ln x = \frac{4}{3}\ln x \ \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 2

attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p \ln x = \frac{\ln x + \frac{1}{3}\ln x}{2}$ ***A1***

$2p \ln x = \frac{4}{3}\ln x \ \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 3

attempt to find difference using u_3 ***M1***

$$\frac{1}{3}\ln x = \ln x + 2d \ \left(\Rightarrow d = -\frac{1}{3}\ln x\right)$$

$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right)$ OR $p \ln x - \ln x = -\frac{1}{3}\ln x$ ***A1***

$p \ln x = \frac{2}{3}\ln x$ ***A1***

$p = \frac{2}{3}$ ***AG***

[3 marks]

(b.ii)

Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$.

[1]

Markscheme
$d = -\frac{1}{3}\ln x$ ***A1***   ***[1 mark]***

(b.iii)

The sum of the first n terms of the series is $-3\ln x$.

Find the value of n.

[6]

Markscheme

METHOD 1

$$S_n = \frac{n}{2}\left[2\ln x + (n-1) \times \left(-\frac{1}{3}\ln x\right)\right]$$

attempt to substitute into S_n and equate to $-3\ln x$ ***(M1)***

$$\frac{n}{2}\left[2\ln x + (n-1) \times \left(-\frac{1}{3}\ln x\right)\right] = -3\ln x$$

correct working with S_n (seen anywhere) ***(A1)***

$\frac{n}{2}\left[2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right]$ OR $n\ln x - \frac{n(n-1)}{6}\ln x$ OR $\frac{n}{2}\left(\ln x + \left(\frac{4-n}{3}\right)\ln x\right)$

correct equation without $\ln x$ ***A1***

$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$ OR $n - \frac{n(n-1)}{6} = -3$ or equivalent

Note: Award as above if the series $1 + p + \frac{1}{3} + \cdots$ is considered leading to $\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$.

attempt to form a quadratic $= 0$ ***(M1)***

$$n^2 - 7n - 18 = 0$$

attempt to solve their quadratic ***(M1)***

$$(n-9)(n+2) = 0$$

$n = 9$ ***A1***

METHOD 2

listing the first 7 terms of the sequence ***(A1)***

$$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \cdots$$

recognizing first 7 terms sum to 0 ***M1***

8^{th} term is $-\frac{4}{3}\ln x$ ***(A1)***

9^{th} term is $-\frac{5}{3}\ln x$ ***(A1)***

sum of 8^{th} and 9^{th} term $= -3\ln x$ ***(A1)***

$n = 9$ ***A1***

[6 marks]

14. 22M.1.SL.TZ1.9

(a.i)

Expand and simplify $(1-a)^3$ in ascending powers of a.

[2]

Markscheme

EITHER

attempt to use binomial expansion ***(M1)***

$$1 + \ \times 1 \times (-a) + \ \times 1 \times (-a)^2 + 1 \times (-a)^3$$

OR

$$(1-a)(1-a)(1-a)$$

$= (1-a)(1-2a+a^2)$ ***(M1)***

THEN

$= 1 - 3a + 3a^2 - a^3$ ***A1***

[2 marks]

(a.ii)

By using a suitable substitution for a, show that $1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x = 8\sin^6 x$.

[4]

Markscheme

$a = \cos 2x$ *(A1)*
So, $1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x =$
$(1 - \cos 2x)^3$ ***A1***
attempt to substitute any double angle rule for $\cos 2x$ into $(1 - \cos 2x)^3$ ***(M1)***
$= (2\sin^2 x)^3$ ***A1***
$= 8\sin^6 x$ ***AG***

Note: Allow working RHS to LHS.

[4 marks]

(b.i)

Show that $\int_0^m f(x)dx = \frac{32}{7}\sin^7 m$, where m is a positive real constant.

[4]

Markscheme

recognizing to integrate $\int (4\cos x \times 8\sin^6 x)dx$ ***(M1)***

EITHER
applies integration by inspection ***(M1)***
$$32\int (\cos x \times (\sin x)^6)dx$$
$= \frac{32}{7}\sin^7 x\ (+c)$ ***A1***
$\left[\frac{32}{7}\sin^7 x\right]_0^m \quad \left(= \frac{32}{7}\sin^7 m - \frac{32}{7}\sin^7 0\right)$ ***A1***

OR
$u = \sin x \Rightarrow \frac{du}{dx} = \cos x$ ***(M1)***
$$\int 32\cos x\,(\sin^6 x)dx = \int 32u^6\,du$$
$= \frac{32}{7}u^7\ (+c)$ ***A1***
$\left[\frac{32}{7}\sin^7 x\right]_0^m$ OR $\left[\frac{32}{7}u^7\right]_0^{\sin m} \quad \left(= \frac{32}{7}\sin^7 m - \frac{32}{7}\sin^7 0\right)$ ***A1***

THEN
$= \frac{32}{7}\sin^7 m$ ***AG***

[4 marks]

(b.ii)

It is given that $\int_m^{\frac{\pi}{2}} f(x)dx = \frac{127}{28}$, where $0 \le m \le \frac{\pi}{2}$. Find the value of m.

[5]

Markscheme

EITHER

$\int_m^{\frac{\pi}{2}} f(x)dx \left(= \left[\frac{32}{7}\sin^7 x\right]_m^{\frac{\pi}{2}}\right) = \frac{32}{7}\sin^7 \frac{\pi}{2} - \frac{32}{7}\sin^7 m$ ***M1***

$\frac{32}{7}\sin^7 \frac{\pi}{2} - \frac{32}{7}\sin^7 m = \frac{127}{28}$ OR $\frac{32}{7}(1 - \sin^7 m) = \frac{127}{28}$ ***(M1)***

OR

$\int_0^{\frac{\pi}{2}} f(x)dx = \int_0^m f(x)dx + \int_m^{\frac{\pi}{2}} f(x)dx$ ***M1***

$\frac{32}{7} = \frac{32}{7}\sin^7 m + \frac{127}{28}$ ***(M1)***

THEN

$\sin^7 m = \frac{1}{128} \left(= \frac{1}{2^7}\right)$ ***(A1)***

$\sin m = \frac{1}{2}$ ***(A1)***

$m = \frac{\pi}{6}$ ***A1***

[5 marks]

15. 25M.2.SL.TZ1.8

(a)

Write down t_3.

[1]

Markscheme

15 ***A1***

[1 mark]

(b)

Find t_4.

[2]

Markscheme

attempt to add 11 cards onto a stack with 3 rows **OR** attempt to consider all 4 rows ***(M1)*** valid diagram with 4 rows **OR** $t_4 = 15 + 11$ **OR** $t_4 = 2 + 5 + 8 + 11 = 26$ ***A1***
[2 marks]

(c)

Show that $t_n = \frac{n(3n+1)}{2}$.

[3]

Markscheme

METHOD 1 recognition that t_n is a sum of an arithmetic sequence ***(M1)*** $t_n = 2 + 5 + 8 + 11 + \ldots$ attempt to use formula for the sum of n terms of an arithmetic sequence ***M1*** $t_n = \frac{n}{2}\left(2(2) + 3(n-1)\right)$ ***A1*** $t_n = \frac{n}{2}(3n+1)$ ***AG*** **METHOD 2** attempt to split t_n into the total number of stacked and horizontal cards ***(M1)***
stacked $2 + 4 + 6 + \ldots = \frac{n}{2}\left(4 + 2(n-1)\right)\left(= \frac{n}{2}(2n+2)\right)$ ***A1***

horizontal $0 + 1 + 2 + \ldots = \frac{n}{2}\left(0 + 1(n-1)\right)\left(= \frac{n}{2}(n-1)\right)$ ***A1***

$t_n = \frac{n}{2}\left(4 + 2(n-1)\right) + \frac{n}{2}\left(0 + 1(n-1)\right)\left(= \frac{n}{2}(2n+2) + \frac{n}{2}(n-1)\right)$ $t_n = \frac{n}{2}(3n+1)$ ***AG*** **METHOD 3** recognition that a stack with n rows is made up of complete triangles with the bottom row of horizontal cards removed and that the numbers of complete triangle cards form an arithmetic sequence ***(M1)*** $t_n = (3 + 6 + 9 + 12 + \ldots + 3n) - n$ **OR** $t_n = 3(1 + 2 + 3 + 4 + \ldots + n) - n$ attempt to use formula for the sum of n terms of an arithmetic sequence ***M1*** $t_n = \frac{n}{2}\left(2(3) + 3(n-1)\right) - n$ **OR** $t_n = 3 \times \frac{n}{2}(1+n) - n$ ***A1*** $t_n = \frac{n}{2}(3n+1)$ ***AG***
[3 marks]

(d)

A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack.

[3]

Markscheme
attempt to solve $\frac{n(3n+1)}{2} \leq 14(52)\ (=728)$ ***(M1)*** **Note:** Accept an attempt to solve an equation for ***(M1)***. $21.8642...$ **OR** $n = 21,\ t_n = 672$ and $n = 22,\ t_n = 737$ ***(A1)*** max number of rows is 21 ***A1*** ***[3 marks]***

(e)

A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack.

[2]

Markscheme
EITHER attempt to solve by listing at least six values of t_n ***(M1)*** $2,\ 7,\ 15,\ 26,\ 40,\ 57...$ **OR** recognition that $\frac{\frac{1}{2}n(3n+1)}{52}$ must be an integer ***(M1)*** $\frac{1}{2}n(3n+1) = 52k$ (where k is an integer) **THEN** min number of rows is 13 ***A1*** **Note:** Award ***(M1)A0*** for an answer of 5 packs. Award ***M0A0*** for any answer resulting from solving $\frac{1}{2}n(3n+1) = 52$. ***[2 marks]***

(f)

Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored.

[5]

Markscheme

EITHER attempt to use Pythagoras's Theorem or trigonometry to find the height of an equilateral triangle with sides $88\,mm$ ***(M1)*** height $=$ $\sqrt{88^2 - 44^2}$ **OR** $88\sin 60°$ **OR** $88\cos 30°$ **OR** $44\tan 60°$ **OR** $\frac{44}{\tan 30°}$ **OR** $44\sqrt{3}(= 76.2102\ldots)$ ***(A1)*** attempt to solve $44n\sqrt{3} >$ $2\,000$ **OR** their perpendicular height$\times\ n > 2\,000$ ***(M1)***

Note: Accept an attempt to solve an equation for ***(M1)***.

OR attempt to use trigonometry to find the side of an equilateral triangle with height $2\,000\,mm$ ***(M1)*** side $= \frac{2\,000}{\sin 60°}$ **OR** $\frac{2\,000}{\cos 30°}$ **OR** $\frac{4\,000}{\sqrt{3}}(=$ $2\,309.40\ldots)$ ***(A1)*** attempt to solve $88n > 2\,309.40\ldots$ **OR** $88n >$ their side ***(M1)***

Note: Accept an attempt to solve an equation for ***(M1)***.

THEN $n > 26.2431\ldots$ so min number of rows is 27 ***(A1)*** $t_{27} =$ $1\,107$ ***A1***
[5 marks]

16. 25M.2.SL.TZ1.9

(a)

Show that $f^{-1}(x) = f(x)$.

[3]

Markscheme

METHOD 1 attempt to rearrange and swap x and y (at any stage) ***(M1)*** $x = \frac{2-2y}{y+2} \Rightarrow xy + 2x = -2y + 2$ $xy + 2y = 2 -$ $2x$ **OR** $y(x+2) = 2 - 2x$ ***A1*** $f^{-1}(x) = \frac{2-2x}{x+2}$ **OR** $y =$ $\frac{2-2x}{x+2}$ ***A1*** So $f^{-1}(x) = f(x)$ ***AG*** **METHOD 2** attempt to find $f\big(f(x)\big)$ ***(M1)***

$f\left(f(x)\right) = \frac{2-2\left(\frac{2-2x}{x+2}\right)}{\left(\frac{2-2x}{x+2}\right)+2} = \frac{2(x+2)-2(2-2x)}{(2-2x)+2(x+2)} \left(= \frac{6x}{6}\right)$ ***A1*** $= x$ ***A1*** So $f^{-1}(x) = f(x)$ ***AG***
[3 marks]

(b.i)

Find the value of k.

[3]

Markscheme

EITHER $OP = \sqrt{k^2 + \left(\frac{2-2k}{k+2}\right)^2}$ ***(A1)*** attempt to minimise distance ***(M1)***
OR recognition that P lies on $y = x$ due to symmetry ***(M1)*** $k = -4.44948...$ or $k = 0.449489...$ ***(A1)*** **THEN** minimum when $k = 0.449489...$ $k = 0.449$ ***A1***

Note: If no working seen, award ***(M0)(A0)A0*** for 0.45.
[3 marks]

(b.ii)

Hence, write down the coordinates of P.

[1]

Markscheme

coordinates of P $(0.449,\ 0.449)$ ***A1***

Note: Award ***A1FT*** for $f(k)$, using their k.
[1 mark]

(c)

In terms of c and d, write down the equation of

[[N/A]]

(c.i)

the vertical asymptote;

[1]

Markscheme
$x = -\frac{d}{c}$ ***A1***   **Note:** Answers in (i) and (ii) must be correct equations.   ***[1 mark]***

(c.ii)

the horizontal asymptote.

[1]

Markscheme
$y = -\frac{3}{c}$ ***A1***   **Note:** Answers in (i) and (ii) must be correct equations.   ***[1 mark]***

(d)

Find the value of d.

[2]

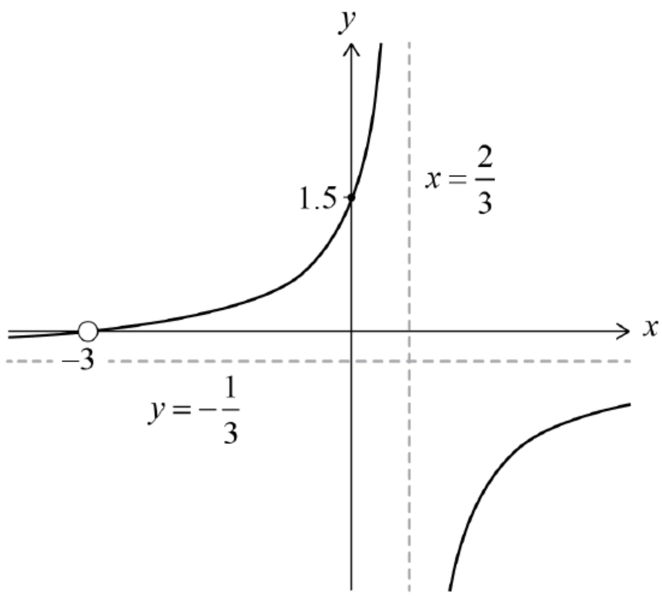
Markscheme
$g^{-1}(x) = \frac{2-dx}{cx+3}$ **OR** $-\frac{d}{c} = -\frac{3}{c}$ ***(A1)*** $d = 3$ ***A1***   ***[2 marks]***

(e)

Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations.

[4]

Markscheme



Only if shape is approximately correct for this rational function (i.e. two branches in opposite corners and displaying appropriate asymptotic behaviour), award the following marks independently. asymptotes $x = \frac{2}{3}$ and $y = -\frac{1}{3}$ drawn with correct labels ***A1A1*** labelled point discontinuity (or x-intercept) at $(-3,\ 0)$, labelled y-intercept at $\left(0,\ \frac{3}{2}\right)$ ***A1A1***

Note: Award ***A1FTA1FT***, as appropriate, for their value of d (i.e. x-intercept= $-d$, y-intercept $\frac{d}{2}$).

[4 marks]

17. 25M.2.SL.TZ2.8

(a)

Find the probability that a randomly chosen apple weighs less than 170 grams.

[2]

Markscheme
recognising to find $P\,(W < 170)$ ***(M1)*** $P\,(W < 170) = 0.265985\ldots$ $P\,(W < 170) = 0.266$ (accept $26.6\,\%$) ***A1*** ***[2 marks]***

(b)

It is found that $20\,\%$ of the apples weigh more than w grams. Find w, correct to four significant figures.

[2]

Markscheme
recognising $P\,(W < w) = 0.2$ **OR** labelled sketch (or equivalent) ***(M1)*** $w = 181.732\ldots$ $w = 181.7$ (grams) (must be 4 sf) ***A1*** ***[2 marks]***

(c)

Find the percentage of apples that are classified as premium at Adam's Apple Orchard.

[2]

Markscheme
recognizing the need to find $P\,(170 < W < 185)$ ***(M1)*** diagram, $P\,(170 < W < 185) = 0.628\,(364\ldots)$ $62.8\,\%$ ***A1*** ***[2 marks]***

(d)

Find the probability that a randomly chosen box contains at least 30 premium apples.

[3]

Markscheme
recognising binomial probability ***(M1)*** $X \sim B(40,\ 0.628364\ldots)$ **OR** $n = 40$ and $p = 0.628364\ldots$ ***(A1)*** $P\ (X \geq 30) = 0.073861\ldots$ $P\ (X \geq 30) = 0.0739$ (accept $7.39\,\%$) ***A1*** ***[3 marks]***

(e)

If 10 of these boxes are randomly selected, find the probability that exactly 4 boxes have at least 30 premium apples.

[2]

Markscheme
$Y \sim B\ (10,\ 0.073861\ldots)$ ***(A1)*** $P\ (Y = 4) = 0.003944\ldots$ $P\ (Y = 4) = 0.00394$ (accept $0.394\,\%$) ***A1*** ***[2 marks]***

(f)

Find the value of μ.

[6]

Markscheme
$P\ (W < 170) = 0.12$ **OR** $P\ (W > 185) = 0.06$ ***(A1)*** use of inverse normal to find z score for their probability ***(M1)*** $z = -1.17(4\,986\ldots),\ z = 1.55(4\,773\ldots)$ ***(A1)(A1)***   attempt to solve their two equations ***(M1)*** $-1.174986\ldots = \frac{170-\mu}{\sigma}$ $1.554773\ldots \frac{185-\mu}{\sigma}$ $\mu = 176.456\ldots$ $\mu = 176$ (grams) ***A1*** ***[6 marks]***

18. 25M.2.SL.TZ2.9

(a)

Find the maximum height of Ball 1 after the 5^{th} bounce.

[3]

Markscheme
recognising geometric sequence with $r = \frac{2}{3}$ (seen anywhere) ***(M1)*** $10\left(\frac{2}{3}\right)^{6-1}$ **OR** $\frac{20}{3}\left(\frac{2}{3}\right)^{5-1}$ ***(A1)*** $1.31687...$ height after 5^{th} bounce= 1.32 (m) (accept $\frac{320}{243}$) ***A1*** ***[3 marks]***

(b)

Find the total distance travelled by Ball 1 immediately before the 5^{th} bounce.

[3]

Markscheme
recognition of the need to use S_n (seen anywhere) ***(M1)*** $10 + 10 \times \left(\frac{2}{3}\right) + 10 \times \left(\frac{2}{3}\right)^2 + 10 \times \left(\frac{2}{3}\right)^3 + 10 \times \left(\frac{2}{3}\right)^4$ **OR** S_5 recognition to double the height ***(M1)*** $10 + 2 \times S_4$ **OR** $2 \times S_5 - 10$ **OR** $10 + 2 \times 10 \times \left(\frac{2}{3}\right) + 2 \times 10 \times \left(\frac{2}{3}\right)^2 + 2 \times 10 \times \left(\frac{2}{3}\right)^3 + 2 \times 10 \times \left(\frac{2}{3}\right)^4$ (or equivalent) $42.0987...$ total distance travelled= 42.1 ***A1*** ***[3 marks]***

(c)

A ball is dropped from a height of x metres. Show that $\delta = 5x$ metres.

[3]

Markscheme
recognising the need to use S_∞ formula (seen anywhere) ***(M1)*** $2S_\infty + u_1$ **OR** $2S_\infty - x$ $2\left(\frac{\frac{2}{3}x}{1-\frac{2}{3}}\right) + x$ **OR** $2\left(\frac{x}{1-\frac{2}{3}}\right) - x$ **OR** $\frac{x}{1-\frac{2}{3}} + \frac{\frac{2}{3}x}{1-\frac{2}{3}}$ ***A1*** $2(2x) + x$ **OR** $6x - x$ **OR** $3x + 2x$ ***A1*** total distance travelled= $5x$ ***AG***   ***[3 marks]***

(d)

Write down the value of δ_1.

[1]

Markscheme
$\delta_1 = 50$ (m) ***A1***   ***[1 mark]***

(e)

Determine which tennis ball is the first ball to travel less than 25 metres.

[4]

Markscheme
$\delta_2 (= 5 \times 9.56) = 47.8$ $d = 47.8 - 50 (= -2.2)$ **OR** $d = 5 \times -0.44 (= -2.2)$ ***(A1)*** attempt to find n using nth term of an AP with their d ***(M1)*** $50 + (n-1)(-2.2) < 25$ (accept equations) $12.3636...$ ***(A1)*** Ball 13 ***A1***   ***[4 marks]***

19. 25M.2.SL.TZ3.8

(a)

State two conditions required for X to be modelled by a binomial distribution.

[2]

Markscheme
fixed number of trials each trial has two possible outcomes ($p,\ q$ with $P\ (p) + P\ (q) = 1$) outcome of each trial is independent of all the others **OR** probability of success constant. ***A1A1*** **Note:** Award ***A1*** for each correct condition. ***[2 marks]***

(b)

Find the number of people that are expected to ride *Daifong*.

[2]

Markscheme
$1\,900 \times 0.37$ ***(A1)*** 703 (people) ***A1*** ***[2 marks]***

(c)

Find the probability that

[[N/A]]

(c.i)

712 people will ride *Daifong*;

[2]

Markscheme
Let D be the number of people who will ride *Daifong* recognition of binomial distribution ***(M1)*** $P\ (D = 712) = 0.0172556\ldots = 0.0173$ ***A1*** ***[2 marks]***

(c.ii)

between 684 and 712 people, inclusive, will ride *Daifong*.

[2]

Markscheme
recognizing the need to find $P\,(684 \leq D \leq 712)$ ***(M1)*** $P\,(D \leq 712) = 0.674739...\ P\,(D \leq 683) = 0.177146...\ P\,(684 \leq D \leq 712) = 0.497593...$ $= 0.498$ ***A1***   **Note:** If the normal approximation to the binomial distribution is used, award marks as appropriate.   ***[2 marks]***

(d)

Given that between 684 and 712 people, inclusive, will ride *Daifong*, find the probability that at most 692 people will ride *Daifong*.

[4]

Markscheme
recognizing conditional probability ***(M1)***   **Note:** Recognition must be shown in context either in words or symbols, not just $P\,(A\ B)$. $P\,(D \leq 692\ 684 \leq D \leq 712)\ \left(= \frac{P\,(684 \leq D \leq 692)}{P\,(684 \leq D \leq 712)}\right)$ or equivalent in words $P\,(684 \leq D \leq 692) = 0.132318...$ (seen anywhere) ***(A1)*** $\left(\frac{P\,(684 \leq D \leq 692)}{P\,(684 \leq D \leq 712)} =\right) \frac{0.132318...}{0.497593...}$ ***(A1)*** $= 0.265917... = 0.266$ ***A1***   ***[4 marks]***

(e)

Find the probability that a person will ride both *Daifong* and *Torbellino*.

[2]

Markscheme

substituting into formula for independent events ***(M1)*** $= 0.37 \times 0.61 =$
0.226 $(= 0.2257$ (exact)) ***A1***
[2 marks]

(f)

Find the value of n.

[3]

Markscheme

(let T be the number of people who went on *Torbellino*) use of tables, guess-and-check or inverse binomial ***(M1)***
Note: Award ***(M1)*** for at least one correct value of $P\,(T \leq 500)$ for a value of n. Award ***(M1)*** for an attempt to set up the normal distribution with $z =$ 0.504.
$n = 809$ (accept $n = 808$) ***A2***
[3 marks]

20. 25M.2.SL.TZ3.9

(a.i)

Write down the value of $v\,(1)$.

[1]

Markscheme

$7.43076\ldots$ $v\,(1) = 7.43\ (m\,s^{-1})$ ***A1***
[1 mark]

(a.ii)

Find the time when Fiona's velocity is $5\,m\,s^{-1}$.

[2]

Markscheme
equating $v\,(t)$ and 5 ***(M1)*** $0.348114...\ 0.348$ (seconds) ***A1***   ***[2 marks]***

(b)

Find the time when Fiona's acceleration is $4\,m\,s^{-2}$.

[2]

Markscheme
recognizing $a = v'\,(t)$ ***(M1)*** $0.590930...\ 0.591$ (seconds) ***A1***   ***[2 marks]***

(c.i)

Write down the limit of $v\,(t)$ as t approaches infinity.

[2]

Markscheme
considering large values of t ***(M1)*** $\left(\lim_{t\to\infty}\left(v\,(t)\right) =\right)\ 8.14\,(m\,s^{-1})$ ***A1***   ***[2 marks]***

(c.ii)

State a reason why the value in part (c)(i) is not valid in the context of this question

[1]

Markscheme

EITHER the race lasts a finite time (e.g. it ends after (Fiona) crosses the line) ***R1*** **OR** the graph approaches the limiting value but Fiona will never attain that speed ***R1*** **OR** Fiona cannot maintain that speed for a long period of time ***R1*** **Note:** Award ***R1*** for a valid reason that does not contradict their answer to part (c)(i).
[1 mark]

(d)

Find the distance Lucy is from the finishing line when Fiona completes the 200 metres.

[6]

Markscheme

Fiona takes t_f seconds to travel $200\,m$
EITHER recognizing distance travelled by either athlete in the first t seconds is $\int_0^t v\ (t)\,dt$ **OR** $\int_0^t w\ (t)\,dt$ (seen anywhere) ***(M1)*** equating distance travelled by Fiona to $200\ (m)$ ***(M1)*** $\int_0^{t_f} v\ (t)\,dt = 200$
OR attempt to integrate $v\ (t)$ ***(M1)*** $s\ (t) = 8.14\sqrt{t^2+0.2} - 3.64031\ldots$ equating 200 to *their* $s\ (t)$ (must include a constant of integration) ***(M1)*** $8.14\sqrt{t^2+0.2} - 3.64031\ldots = 200$ **THEN** $t_f = 25.0132\ldots$ (accept 25) ***(A1)*** recognition that a definite integral of $w\ (t)$ with 0 and *their* t_f is required ***(M1)*** (Lucy's distance $=$) $\int_0^{25.0132\ldots} w\ (t)\,dt$ $(= 195.772\ldots)$ distance from finishing line $=$ $200\,-$*their* Lucy's distance ***(M1)*** $4.22788\ldots$ $4.23\ (m)$ ***A1***
[6 marks]

21. 24N.2.SL.TZ1.8

(a.i)

Write down the value of a and the value of b.

[2]

Markscheme
$a = 0.362$ (exact); $b = 30.5$ (exact) ***A1A1*** **Note**: Award ***A1A0*** if the values of a and b are interchanged or not labelled. ***[2 marks]***

(a.ii)

Interpret, in context, the value of a.

[1]

Markscheme
a represents the (average) rate of increase (change) in population (0.362 millions of people per year). (or equivalent) ***R1*** ***[1 mark]***

(b)

Comment on the reliability of the student's prediction.

[1]

Markscheme
It is unreliable because 2030 is outside the range of data (extrapolation). ***A1*** ***[1 mark]***

(c)

[N/A]

[[N/A]]

(c.i)

Use Benoit's model to predict the population of Canada in the year 2100.

[2]

Markscheme
attempt to find $B(100)$ ***(M1)*** $61.4707\ldots\ 61.5$ million OR $61{,}500{,}000$ ***A1***   ***[2 marks]***

(c.ii)

Interpret, in context, the value 1.007 in Benoit's model.

[1]

Markscheme
The annual growth rate of the population is $0.7\ \%$. ***A1*** **Note**: Description must include some reference to annual rate.   ***[1 mark]***

(d)

Use Cecilia's model to predict the population of Canada in the year 2100.

[1]

Markscheme
$58.1070\ldots\ 58.1$ million OR $58{,}100{,}000$ ***A1***   ***[1 mark]***

(e)

Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest.

[3]

Markscheme

consideration of the difference function $C(t) - B(t)$ or $B(t) - C(t)$ or $|C(t) - B(t)|$ ***(M1)*** evidence of finding the maximum (or minimum) of this function. ***(M1)*** $t = 45.9583\ 2045$ (accept 2046) ***A1***

[3 marks]

(f)

Find the value of

[[N/A]]

(f.i)

$$B'(40);$$

[1]

Markscheme
$0.282151 \ldots B'(40) = 0.282$ ***A1***   ***[1 mark]***

(f.ii)

$$C'(40).$$

[1]

Markscheme
$0.325546 \ldots C'(40) = 0.326$ ***A1***   ***[1 mark]***

(g)

Compare and interpret, in context, the values of $B'(40)$ and $C'(40)$.

[2]

Markscheme
$B'(40) < C'(40)$ (or equivalent in words) ***A1*** the population in Cecilia's model is increasing at a faster rate than in Benoit's model (in 2040) (or equivalent) ***R1*** **Note**: Do not award ***A0R1***. ***[2 marks]***

22. 24N.2.SL.TZ1.9

(a.i)

Write down the amplitude of the graph of f.

[1]

Markscheme
3 ***A1*** ***[1 mark]***

(a.ii)

Find the period of f.

[2]

Markscheme
attempt to find period ***(M1)*** $\frac{2\pi}{b}$ or $\frac{2\pi}{4\pi}$ period $= \frac{1}{2}$ ***A1*** ***[2 marks]***

(b)

By considering the graph of $y = f(x) + g(x)$, determine

[[N/A]]

(b.i)

the value of a;

[2]

Markscheme

evidence of considering the graph of $3\sin(4\pi x) - 4\cos(4\pi x)$ (seen in i, ii,or iii) ***(M1)*** $a = 5$ ***A1***
[2 marks]

(b.ii)

the value of b;

[1]

Markscheme

$b = 4\pi$ ***A1***
[1 mark]

(b.iii)

the smallest possible value of c.

[1]

Markscheme

$0.198792 \ldots c = 0.199$ ***A1***
[1 mark]

(c)

Find the time at which the car first reaches its maximum velocity.

[1]

Markscheme

19 (seconds) ***A1***

[1 mark]

(d)

Find the number of speed bumps the car passes over in the first two minutes of motion.

[1]

Markscheme
5 (speed bumps) ***A1*** ***[1 mark]***

(e.i)

Find $v'(t)$.

[2]

Markscheme
attempt to use chain rule (multiplication by $\frac{\pi}{14}$) ***(M1)*** $(3.5)\left(\frac{\pi}{14}\right)\sin\left(\frac{\pi}{14}(t-5)\right)$ OR $0.785398\ldots$ $v'(t) = 0.785\sin\left(\frac{\pi}{14}(t-5)\right)\ \left(= \frac{\pi}{4}\sin\left(\frac{\pi}{14}(t-5)\right)\right)$ ***A1*** ***[2 marks]***

(e.ii)

Hence, or otherwise, write down the maximum acceleration of the car.

[2]

Markscheme

recognition that $v' = a$ *(M1)* $0.785\ (m\,s^{-2})$ $\left(= \frac{\pi}{4}\right)$ ***A1***
[2 marks]

(f)

Find the distance, in metres, between consecutive speed bumps.

[3]

Markscheme

recognition that a definite integral of the velocity function is needed ***(M1)*** using a correct set of limits (any limits which differ by 28 seconds) ***(A1)*** $\int_5^{33} v\,(t)dt$ $\left(= \int_k^{k+28} |v(t)|\,dt\right)$
$252\ \ (m)$ (exact) ***A1***
[3 marks]

23. 24M.2.SL.TZ2.8

(a)

Write down an expression for A in terms of x and h.

[2]

Markscheme

let A_R denote the area of the rectangle and A_S denote the area of the semicircle
one correct area $A_R = 2xh$ OR $A_S = \frac{1}{2}\pi x^2$ ***(A1)*** $A = 2xh + \frac{1}{2}\pi x^2$ $\left(= x\left(2h + \frac{1}{2}\pi x\right)\right)$ ***A1***
[2 marks]

(b)

Given that $P = 10$, show that $h = \frac{1}{2}(10 - 2x - \pi x)$.

[2]

Markscheme
attempts to find a correct expression for the total perimeter ***(M1)*** $(P=)2x+2h+\pi x$, base + 2 ×height + half circumference $2x+2h+\pi x=10$ OR $2h=10-2x-\pi x$ ***A1*** $h=\frac{1}{2}(10-2x-\pi x)$ ***AG***   ***[2 marks]***

(c)

Show that the amount of light, L units, let in by the window is given by $L=30x-6x^2-\frac{5}{2}\pi x^2$.

[4]

Markscheme
$L=3(2xh)+1\left(\frac{1}{2}\pi x^2\right)$ ***(M1)(A1)*** **Note:** Award ***(M1)*** for multiplying $2xh$ by 3 and award ***(A1)*** for a fully correct expression. substitutes $h=\frac{1}{2}(10-2x-\pi x)$ into their expression for L ***M1*** $L=6x\left(\frac{1}{2}(10-2x-\pi x)\right)+\frac{1}{2}\pi x^2=30x-6x^2-3\pi x^2+\frac{1}{2}\pi x^2$ $\left(=30x-\left(6+\frac{5\pi}{2}\right)x^2\right)$ ***A1*** $L=30x-6x^2-\frac{5}{2}\pi x^2$ ***AG***   ***[4 marks]***

(d.i)

Find an expression for $\frac{dL}{dx}$.

[2]

Markscheme
$\frac{dL}{dx}=30-12x-5\pi x$ $(=30-(12+5\pi)x)$ (accept $\frac{dL}{dx}=30-27.7x$) ***(M1)A1***   ***[2 marks]***

(d.ii)

Find the value of x so that the window lets in the maximum amount of light. Justify that this value of x gives a maximum.

[3]

Markscheme

recognition that $\frac{dL}{dx} = 0$ (may be represented graphically) ***(M1)*** $x = 1.08272\ldots$ $x = 1.08 \left(= \frac{30}{12+5\pi}\right)$ (m) ***A1*** correct reasoning to justify a maximum ***R1*** L is a quadratic (function of x) with a negative coefficient of x^2 (may be represented as a sketch indicating maximum point) OR a clearly labelled sign diagram showing the change in gradient OR $\frac{d^2L}{dx^2} = -12 - 5\pi\ (= -27.7079\ldots)(< 0)$

[3 marks]

(d.iii)

Find the value of h so that the window lets in the maximum amount of light.

[2]

Markscheme

attempts to substitute their value of x into h ***(M1)*** $h = 2.21654\ldots$ $h = 2.22 \left(= \frac{30+10\pi}{12+5\pi}\right)$ (m) ***A1***

[2 marks]

24. 24M.2.SL.TZ2.9

(a)

Show that $AB = 28.57$, correct to four significant figures.

[3]

Markscheme

METHOD 1 let M be the midpoint of $[AB]$ and so $AB = 2AM$ attempts to use Pythagoras' theorem to find AM^2 OR AM ***(M1)*** $AM^2 = 20^2 - 14^2\ (= 204)$ OR $AM = \sqrt{20^2 - 14^2}\ (= 14.2828\ldots = \sqrt{204} = 2\sqrt{51})$ recognizes that $AB = 2AM$ ***(A1)*** $AB = 2 \times 14.2828\ldots\ (= 28.5657\ldots)\ (= 2\sqrt{204} = 4\sqrt{51})$ ***A1*** $AB = 28.5657\ldots$ $AB = 28.57$ (m) ***AG*** **METHOD 2** let M be the midpoint of $[AB]$ and so $AB = 2AM$ let $\theta = A\hat{S}M$ $\theta = 0.795398\ldots \left(= \cos^{-1}\frac{14}{20}\right)$ ***(A1)*** attempts to use a valid trigonometric ratio ***(M1)*** **EITHER** $AM = 14\tan(0.795398\ldots) \left(= 14.2828\ldots = 14\tan\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1*** **OR** $AM = 20\sin(0.795398\ldots) \left(= 14.2828\ldots = 20\sin\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1*** **THEN** $AB = 28.5657\ldots$ $AB = 28.57$ (m) ***AG***
[3 marks]

(b)

Show that the sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second.

[1]

Markscheme

EITHER the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second ***A1*** **OR** $\left(\frac{2\pi}{n} = 16 \Rightarrow n =\right)\frac{2\pi}{16}\left(= \frac{\pi}{8}\right)$ ***A1*** **THEN** sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second ***AG***
[1 mark]

(c)

Find the value of T.

[4]

Markscheme

Note: For candidates that used Method 2 in part (a) apply full **FT** from their value of θ. attempts to find 2θ where $\theta = A\hat{S}M$ ***(M1)*** $=$

$2(0.795398\ldots)$ $\left(1.59079\ldots = 2\cos^{-1}\frac{14}{20}\right)$ uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving T ***(M1)*** $\frac{2\pi}{16} = \frac{1.59079\ldots}{T}$ $\left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T}\right)$ ***(A1)*** $T = 4.05093\ldots$ $\left(= \frac{1.59079\ldots}{\frac{2\pi}{16}}\right)$ $\left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}}\right)$ $T = 4.05$ (s) ***A1***

[4 marks]

(d)

Write down an expression for α in terms of t.

[1]

Markscheme
$\alpha = \frac{\pi t}{8}$ ***A1***   ***[1 mark]***

(e)

By using the sine rule in ΔASD, show that the distance, d, at time t, can be modelled by

$$d(t) = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}.$$

[3]

Markscheme
applies sine rule in ΔASD ***A1*** $\frac{d}{\sin\alpha} = \frac{20}{\sin A\hat{D}S}$ attempts to find $A\hat{D}S$ in terms of α ***M1*** $A\hat{D}S = \pi - \beta - \alpha$ $(= \pi - 0.7754 - \alpha)(= 2.366\ldots - \alpha)$ $(= 2.37 - \alpha)$ $d = \frac{20\sin\alpha}{\sin(2.366\ldots - \alpha)}$ $\left(= \frac{20\sin\alpha}{\sin(2.37-\alpha)}\right)$ (accept $d = \frac{20\sin\alpha}{\sin(\pi-\beta-\alpha)}$) ***A1*** $d = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 = \frac{\pi t}{8}\right)}$ ***AG***   ***[3 marks]***

(f)

At $t = 0$, state how far south the turtle is from A.

[1]

Markscheme
18 (m) ***A1*** ***[1 mark]***

(g.i)

Use the expressions for $g(t)$ and $d(t)$ to write down an expression for w in terms of t.

[1]

Markscheme
$w = \left\| 0.05t^2 + 1.1t + 18 - \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)} \right\|$ ***A1*** ***[1 mark]***

(g.ii)

Hence find when and where on the path the water first reaches the turtle.

[3]

Markscheme
attempts to solve $w = 0$ for t ***(M1)*** $t = 3.34880 \ldots (12.7765 \ldots)$ $t = 3.35$ (s) ***A1*** $22.2444 \ldots$ 22.2 (m) (south of A) ***A1*** ***[3 marks]***

25. 23M.2.SL.TZ2.8

(a)

Find the period of the function.

[2]

Markscheme
$7.8 = \frac{2\pi}{period}$ ***(M1)*** $\frac{2\pi}{8} = 0.805536 \ldots$ $period = 0.806$ $\left(= \frac{20\pi}{78}\right)$ ***A1*** ***[2 marks]***

(b)

Find the value of

[[N/A]]

(b.i)

a;

[2]

Markscheme
METHOD 1 $amplitude = \frac{\max - \min}{2}$ ***(M1)*** $\frac{1.8-1}{2}$ $a = 0.4$ ***A1*** **METHOD 2** (applies to both (b)(i) and (b)(ii).) attempt to form two simultaneous equations in a and b ***(M1)*** $H(0) = 1 \Rightarrow a + b = 1,\ H\left(\frac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$ $a = -0.4,\ b = 1.4$ ***A1A1*** ***[2 marks]***

(b.ii)

b.

[1]

Markscheme
METHOD 1 $b = 1.4$ ***A1*** **METHOD 2** (applies to both (b)(i) and (b)(ii).) attempt to form two simultaneous equations in a and b ***(M1)*** $H(0) = 1 \Rightarrow a + b = 1,\ H\left(\frac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$ $a = -0.4,\ b = 1.4$ ***A1A1*** ***[1 mark]***

(c)

Find the number of times that the weight reaches its maximum height in the first five seconds of its motion.

[2]

Markscheme
EITHER $\frac{5}{period} = 6.207 \ldots < 6\frac{1}{2}$ ***(A1)*** **OR** consideration of number of maximums on graph in first 5 seconds ***(A1)*** **OR** maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ ***(A1)*** **THEN** 6 times ***A1 [2 marks]***

(d)

Find the first time that the base of the weight reaches a height of 1.5 metres.

[2]

Markscheme
recognizing that $H(t) = 1.5$ ***(M1)*** $-0.4\cos(7.8\,t) + 1.4 = 1.5$ $0.233779 \ldots\ t = 0.234$ (seconds) ***A1 [2 marks]***

(e)

Find the probability that the height of the base of the weight is greater than 1.5 metres at the time the picture is taken

[4]

Markscheme
finding second time height is 1.5 metres ***(M1)*** $t = 0.571757 \ldots$ in each period, height is greater than 1.5 metres for 0.337978… seconds ***(A1)*** **Note:** Award ***(M1)(A1)*** for total time $2.02787 \ldots$ seen. multiplying their value by 6 and divide by 5 ***(M1)*** $\frac{0.337978\ldots\times 6}{5}$ OR

$\frac{2.02787...}{5} = 4.05574 ... P(height\ is\ greater\ than\ 1.5\,m) = 0.406$ ***A1 [4 marks]***

26. 23M.2.SL.TZ2.9

(a)

Find, in terms of n, the probability that

[[N/A]]

(a.i)

the first ball drawn is green;

[1]

Markscheme
$\frac{10}{n}$ ***A1 [1 mark]***

(a.ii)

the first two balls are green.

[2]

Markscheme
multiplying probabilities for *GG* ***(M1)*** $P(GG) = \frac{10}{n} \times \frac{9}{n-1}\ P(GG) = \frac{90}{n^2-n}\ \left(accept\ \frac{90}{n(n-1)}\right)$ ***A1 [2 marks]***

(b)

Show that the probability that the first two balls are red is 0.35.

[2]

Markscheme
$P(First\ red) = \frac{15}{25}$ and $P(Second\ red) = \frac{14}{24}$ (seen anywhere) ***(A1)*** $P(RR) = \frac{15}{25} \times \frac{14}{24}$ (or equivalent) ***A1*** $= 0.35$ ***Ag [2 marks]***

(c)

Find the probability that the first three balls are all red.

[2]

Markscheme
$\frac{15}{25} \times \frac{14}{24} \times \frac{13}{23}$ OR $0.35 \times \frac{13}{23}$ ***(A1)*** $0.197826 \ldots$ $P(three\ red) = 0.198$ (exact answer is $\frac{91}{460}$) ***A1 [2 marks]***

(d)

Find the probability that at least one of the first three balls is green.

[2]

Markscheme
$P(at\ least\ one\ green) = 1 - P(three\ red)$ OR $P(at\ least\ one\ G) = P(one\ G) + P(two\ G) + P(three\ G)$ ***(M1)*** $1 - \left(\frac{15}{25} \times \frac{14}{24} \times \frac{13}{23}\right)$ OR $3\left(\frac{10}{25} \times \frac{15}{24} \times \frac{14}{23}\right) + 3\left(\frac{10}{25} \times \frac{9}{24} \times \frac{15}{23}\right) + \left(\frac{10}{25} \times \frac{9}{24} \times \frac{8}{23}\right)$ $0.802173 \ldots$ $P(at\ least\ one\ green) = 0.802$ (exact answer is $\frac{369}{460}$) ***A1 [2 marks]***

(e)

Find the least value of k such that Millie's expected score is greater than 100.

[6]

Markscheme

$P(first\ green\ on\ third\ draw) = \frac{15}{25} \times \frac{14}{24} \times \frac{10}{23} \times \frac{22}{22} \left(= \frac{7}{46} = 0.152173 \ldots\right)$ ***(A1)*** $P(first\ green\ on\ fourth\ draw) = \frac{15}{25} \times \frac{14}{24} \times \frac{13}{23} \times \frac{10}{22} \left(= \frac{91}{1012} = 0..0899209 \ldots\right)$ ***(A1)*** **Note:** The first two ***(A1)*** are independent. attempt to substitute their probabilities into expected value formula ***(M1)*** expected points per game $= 10 \times \frac{7}{46} + 50 \times \frac{91}{1012} \left(= \frac{3045}{506} = 6.01778 \ldots\right)$ ***(A1)*** setting up inequality or equation in k ***(M1)*** $\frac{3045}{506} k > 100$ $k > 16.6174 \ldots \left(= \frac{10120}{609}\right)$ Millie must play at least 17 times. ***A1*** ***[6 marks]***

27. 22N.2.SL.TZ0.8

(a)

Show that the area of the quadrilateral ADOE is $\frac{r^2}{\tan \alpha}$.

[4]

Markscheme

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

attempt to use right angled trigonometry or sine rule to find AE in terms of r and α ***(M1)***

$\tan \alpha = \frac{r}{\text{AE}}$ OR $\frac{\text{AE}}{\sin\left(\frac{\pi}{2} - \alpha\right)} = \frac{r}{\sin \alpha}$

$\text{AE} = \frac{r}{\tan \alpha}$ OR $\text{AE} = \frac{r \sin\left(\frac{\pi}{2} - \alpha\right)}{\sin \alpha}$ OR $\text{AE} = \frac{r \cos \alpha}{\sin \alpha}$ ***A1***

valid approach to find the area of ADOE ***(M1)***

2 × area of triangle AOE OR area of triangle AED + area of triangle OED OR OE × AE

Area ADOE $= 2\left(\frac{1}{2} \cdot \frac{r}{\tan \alpha} \cdot r\right)$ OR $r \times$ AE ***A1***

Area ADOE $= \frac{r^2}{\tan \alpha}$ ***AG***

[4 marks]

(b.i)

Find $\text{D}\hat{\text{O}}\text{E}$ in terms of α.

[2]

Markscheme

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

recognizing that the sum of the angles of a kite is 2π ***(M1)***
$\text{D}\hat{\text{O}}\text{E} + \text{O}\hat{\text{E}}\text{A} + \text{E}\hat{\text{A}}\text{D} + \text{A}\hat{\text{D}}\text{O} = 2\pi$ OR $2\alpha + 2 \cdot \frac{\pi}{2} + \text{D}\hat{\text{O}}\text{E} = 2\pi$
$\text{D}\hat{\text{O}}\text{E} = \pi - 2\alpha$ ***A1***

Note: Award ***M1A0*** if candidate uses degrees (i.e. $\text{D}\hat{\text{O}}\text{E} + \text{O}\hat{\text{E}}\text{A} + \text{E}\hat{\text{A}}\text{D} + \text{A}\hat{\text{D}}\text{O} = 360°$ or $2\alpha + 2 \cdot \frac{\pi}{2} + \text{D}\hat{\text{O}}\text{E} = 360°$) and obtains $\text{D}\hat{\text{O}}\text{E} = 180° - 2\alpha$.

[2 marks]

(b.ii)

Hence or otherwise, find an expression for the area of R.

[3]

Markscheme

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

valid approach to find the area of R ***(M1)***
area of kite − area of sector OR 2(area of triangle AOE $-$ 0.5 area of sector OED)
Area of sector $= \frac{1}{2}r^2.\text{D}\hat{\text{O}}\text{E}\left(= \frac{1}{2}r^2(\pi - 2\alpha)\right)$ seen anywhere ***(A1)***
Area of R $= \frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha)$ ***A1***

Note: Accept $\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2.\text{D}\hat{\text{O}}\text{E}$.

[3 marks]

(c)

Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r.

[4]

Markscheme

equating their area formula to πr^2 ***(M1)***

$$\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha) = \pi r^2$$

correct equation in terms of α ***A1***

$$\frac{1}{\tan\alpha} - \frac{1}{2}(\pi - 2\alpha) = \pi$$

valid approach to solve the equation ***(M1)***

$$\alpha = 0.218979\ldots$$

$\alpha = 0.219$ ***A1***

[4 marks]

28. 22N.2.SL.TZ0.9

(a)

Find the probability that an employee selected at random works more than 40 hours per week.

[2]

Markscheme

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for P$(T < 55\, T > 40)$ accept P$(T \leq 55\, T > 40)$, P$(T \leq 55\, T \geq 40)$, etc.

recognising to find P$(T > 40)$ ***(M1)***

$$\mathrm{P}(T > 40) = 0.574136\ldots$$

P$(T > 40) = 0.574$ ***A1***

[2 marks]

(b)

A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week.

[3]

Markscheme

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for P$(T < 55 T > 40)$ accept P$(T \leq 55 T > 40)$, P$(T \leq 55 T \geq 40)$, etc.

attempt to multiply four independent probabilities using their P$(T > 40)$ and P$(T < 40)$ ***(M1)***
$(1 - p)^3 \cdot p$ OR $(1 - 0.574136 \ldots)^3 \cdot 0.574136 \ldots$ OR $(0.425863 \ldots)^3 \cdot 0.574136 \ldots$ ***(A1)***

$0.0443430 \ldots$

0.0443 , 0.0444 from 3 sf values ***A1***

[3 marks]

(c.i)

An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week.

[4]

Markscheme

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for P$(T < 55 T > 40)$ accept P$(T \leq 55 T > 40)$, P$(T \leq 55 T \geq 40)$, etc.

recognizing conditional probability ***(M1)***

P$(T < 55 T > 40)$

Note: Award ***(M1)*** for an expression or description in context.
Accept P$(T > 40 T < 55)$ but do not accept just P$(A B)$.

$\frac{P(40<T<55)}{P(T>40)}$ ***(A1)***

$\frac{0.461944...}{0.574136...}$ ***(A1)***

$P(T < 55 T > 40) = 0.804590 ...$

$= 0.805$ ***A1***

[4 marks]

(c.ii)

Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week.

[3]

Markscheme

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for $P(T < 55 T > 40)$ accept $P(T \leq 55 T > 40)$, $P(T \leq 55 T \geq 40)$, etc.

recognizing binomial probability ***(M1)***

$X \sim B(n, p)$

$n = 10$ and $p = 0.804589 ...$ ***(A1)***

$0.0242111 ..., 0.0240188 ...$ using $p = 0.805$

$P(X = 5) = 0.0242$ ***A1***

[3 marks]

(d)

Find the maximum time, in hours per week, that an employee can work and still be considered part-time.

[4]

Markscheme

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for $P(T < 55 T > 40)$ accept $P(T \leq 55 T > 40)$, $P(T \leq 55 T \geq 40)$, etc.

Let P$(T < a) = x$
recognition that probabilities sum to 1 (seen anywhere) ***(M1)***

EITHER
expressing the three regions in one variable ***(M1)***
$x + 0.904 + 2x$ OR P$(T < a) + 0.904 + 2$P$(T < a)$ OR $\frac{1}{2}$P$(T > b) +$ $0.904 +$ P$(T > b)$ OR x and $2x$ correctly indicated on labelled bell diagram
P$(T < a) + 0.904 + 2$P$(T < a) = 1$ OR $\frac{1}{2}$P$(T > b) + 0.904 +$ P$(T > b) =$ 1 (or equivalent) ***(A1)***

OR
expressing either P$(T < a)$ or P$(T > b)$ only in terms of P$(a \leq T \leq b)$ ***(M1)***
(P$(T < a) =$)$\frac{1}{3}(1 -$ P$(a \leq T \leq b))$ OR (P$(T > b) =$)$\frac{2}{3} \cdot (1 -$ P$(a \leq T \leq b))$
$x = \frac{1}{3}(1 - 0.904)(= 0.032)$ OR P$(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$ ***(A1)***

THEN

P$(T < a) = 0.032$
$a = 22.18167\ldots$

$a = 22.2$ accept 22.1 ***A1***

[4 marks]

29. 22M.2.SL.TZ1.8

(a.i)

write down the equation of the vertical asymptote.

[1]

Markscheme
$x = -4$ ***A1***
[1 mark]

(a.ii)

find the equation of the horizontal asymptote.

[2]

Markscheme

attempt to substitute into $y = \frac{a}{c}$ OR table with large values of x OR sketch of f showing asymptotic behaviour ***(M1)***

$y = 4$ ***A1***

[2 marks]

(b.i)

Find $f^{-1}(x)$.

[4]

Markscheme

$$y = \frac{4x+1}{x+4}$$

attempt to interchange x and y (seen anywhere) ***M1***

$xy + 4y = 4x + 1$ OR $xy + 4x = 4y + 1$ ***(A1)***

$xy - 4x = 1 - 4y$ OR $xy - 4y = 1 - 4x$ ***(A1)***

$f^{-1}(x) = \frac{1-4x}{x-4}$ (accept $y = \frac{1-4x}{x-4}$) ***A1***

[4 marks]

(b.ii)

Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis.

[4]

Markscheme

reflection in y-axis given by $f(-x)$ ***(M1)***

$f(-x) = \frac{-4x+1}{-x+4}$ ***(A1)***

reflection of their $f(-x)$ in x-axis given by $-f(-x)$ accept "now $-f(x)$" ***M1***

$$(-f(-x) =) \ -\frac{-4x+1}{-x+4}$$

$= \frac{-4x+1}{x-4}$ OR $\frac{4x-1}{-x+4}$ ***A1***

$= \frac{1-4x}{x-4}$ $\left(= f^{-1}(x)\right)$ ***AG***

Note: If the candidate attempts to show the result using a particular coordinate on the graph of f rather than a general coordinate on the graph of f, where appropriate, award marks as follows:
***M0A0* for eg** $(2,3) \to (-2,3)$
***M0A0* for** $(-2,3) \to (-2,-3)$

[4 marks]

(c.i)

Find the value of p and the value of q.

[2]

Markscheme

attempt to solve $f(x) = f^{-1}(x)$ using graph or algebraically ***(M1)***
$p = -1$ AND $q = 1$ ***A1***

Note: Award ***(M1)A0*** if only one correct value seen.

[2 marks]

(c.ii)

Hence, find the area enclosed by the graph of f and the graph of f^{-1}.

[3]

Markscheme

attempt to set up an integral to find area between f and f^{-1} ***(M1)***
$\int_{-1}^{1}\left(\frac{4x+1}{x+4} - \frac{1-4x}{x-4}\right)dx$ ***(A1)***

$$= 0.675231 \ldots$$

$= 0.675$ ***A1***

[3 marks]

30. 22M.2.SL.TZ1.9

(a)

Find the probability that a randomly selected chocolate muffin weighs less than 61 g.

[2]

Markscheme

P$(C < 61)$ ***(M1)***

$= 0.365112 \ldots$

$= 0.365$ ***A1***

[2 marks]

(b)

In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g.

[2]

Markscheme

recognition of binomial eg $X \sim$ B$(12,\ 0.365 \ldots)$ ***(M1)***

P$(X = 5) = 0.213666 \ldots$

$= 0.214$ ***A1***

[2 marks]

(c.i)

Find the probability that the randomly selected muffin weighs less than 61 g.

[4]

Markscheme

Let CM represent 'chocolate muffin' and BM represent 'banana muffin'
P$(B < 61) = 0.0197555..$ ***(A1)***

EITHER
P$(CM) \times$ P$(C < 61\ CM) +$ P$(BM) \times$ P$(B < 61\ BM)$ (or equivalent in words) ***(M1)***

OR
tree diagram showing two ways to have a muffin weigh < 61 ***(M1)***

THEN
$(0.6 \times 0.365 \ldots) + (0.4 \times 0.0197 \ldots)$ ***(A1)***
$= 0.226969 \ldots$
$= 0.227$ ***A1***

[4 marks]

(c.ii)

Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate.

[3]

Markscheme
recognizing conditional probability ***(M1)*** **Note:** Recognition must be shown in context either in words or symbols, not just P$(A\ B)$ $\frac{0.6 \times 0.365112\ldots}{0.226969\ldots}$ ***(A1)*** $= 0.965183 \ldots$ $= 0.965$ ***A1*** ***[3 marks]***

(d)

Find the value of σ.

[5]

Markscheme

METHOD 1

$\text{P}(CM) \times \text{P}(C < 61\ CM) \times \text{P}(BM) \times \text{P}(B < 61\ BM) = 0.157$ ***(M1)***

$$\big(0.6 \times \text{P}(C < 61)\big) + (0.4 \times 0.0197555 \ldots) = 0.157$$

$\text{P}(C < 61) = 0.248496 \ldots$ ***(A1)***

attempt to solve for σ using GDC ***(M1)***

Note: Award ***(M1)*** for a graph or table of values to show their $\text{P}(C < 61)$ with a variable standard deviation.

$$\sigma = 1.47225 \ldots$$

$\sigma = 1.47$ (g) ***A2***

METHOD 2

$\text{P}(CM) \times \text{P}(C < 61\ CM) \times \text{P}(BM) \times \text{P}(B < 61\ BM) = 0.157$ ***(M1)***

$$\big(0.6 \times \text{P}(C < 61)\big) + (0.4 \times 0.0197555 \ldots) = 0.157$$

$\text{P}(C < 61) = 0.248496 \ldots$ ***(A1)***

use of inverse normal to find z score of their $\text{P}(C < 61)$ ***(M1)***

$$z = -0.679229 \ldots$$

correct substitution ***(A1)***

$$\frac{61 - 62}{\sigma} = -0.679229 \ldots$$

$$\sigma = 1.47225 \ldots$$

$\sigma = 1.47$ (g) ***A1***

[5 marks]