SL AA PAPER 1 (Questions 8– 9) no GDC

1. 25M.1.SL.TZ1.8

The function f is defined by $f(x) = 5(x+1)(x+3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x-h^2)+k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}, x > 0$.

(d.i) Write down an expression for $(f \circ g)(x)$. [1]

(d.ii) Solve the inequality $(f \circ g)(x) \leq 40$. [2]

2. 25M.1.SL.TZ1.9

A solid cylinder has height h cm and base radius R cm.

The cylinder fits exactly inside a hollow sphere of radius r cm.

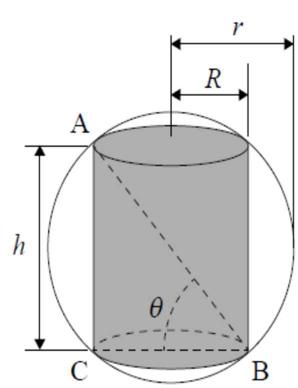
Points A, B and C are points where the surface of the cylinder touches the surface of the sphere.

The line segment $[AB]$ is a diameter of the sphere.

The line segment $[BC]$ is a diameter of the base of the cylinder and $A\hat{B}C = \theta$.

This information is shown on the following diagram.

diagram not to scale



(a.i) By considering triangle ABC, show that $R = r\cos\theta$. [2]

(a.ii) Find an expression for h in terms of r and θ. [2]

(b) Hence or otherwise, show that the total surface area, S cm^2, of the cylinder is given by $S = 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$. [4]

The external surface area of the sphere is $2S$.

(c) Show that $\tan\theta = 2$. [4]

The volume of the cylinder is V cm^3.

(d) Find V, giving your answer in the form $p\pi r^3\sqrt{5}$, where $p \in \mathbb{Q}^+$. [5]

3. 24N.1.SL.TZ1.8

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(a.i) $f(2)$; [2]

(a.ii) $f\left(\frac{1}{8}\right)$. [1]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied.

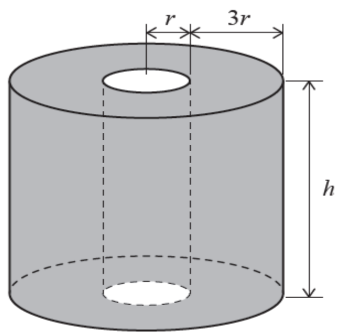
[6]

4. 24N.1.SL.TZ1.9

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b)The total surface area of the hollow cylinder is 240π cm^2.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{dV}{dr}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$.

[3]

5. 24M.1.SL.TZ1.8

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the number of solutions of the equation $f(x) = g(x)$. [2]

6. 24M.1.SL.TZ1.9

A bag contains buttons which are either red or blue.

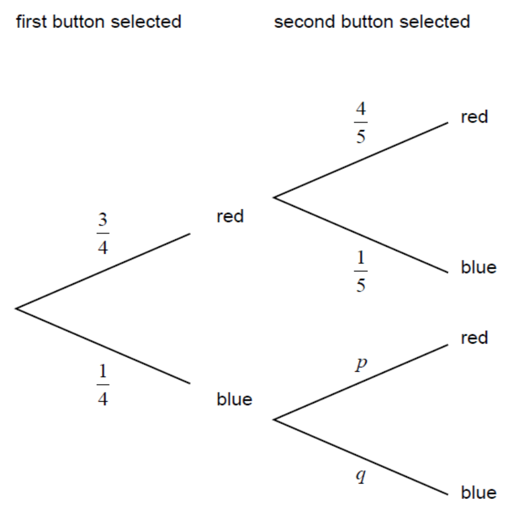
Initially, the bag contains three red buttons and one blue button.

Francine randomly selects one button from the bag. She then replaces the button and adds one extra button of the same colour.

For example, if she selects a red button, she then replaces it and adds one extra red button so that the bag then contains four red buttons and one blue button.

Francine then randomly selects a second button from the bag.

The following tree diagram represents the probabilities of the first two selections.



(a) Find the value of p and the value of q. [2]

(b) Show that the probability that Francine selects two buttons of the same colour is $\frac{7}{10}$. [2]

(c) Given that Francine selects two buttons of the same colour, find the probability that she selects two red buttons. [3]

The random variable X is defined as the number of red buttons selected by Francine.

The following table shows the probability distribution of X.

x	0	1	2
$P(X=x)$	$\frac{1}{10}$	a	b

(d) Find the value of a and the value of b. [2]

(e) Hence, find the expected number of red buttons selected by Francine. [2]

Francine restarts the process with three red buttons and one blue button in the bag. She selects buttons as before, replacing the button and adding one extra button of the same colour each time. She repeats this until she selects a blue button.

(f) Given that the first two buttons she selects are red, write down the probability that the next button she selects is blue. [1]

The probability that she selects the first blue button after n selections in total is $\frac{3}{56}$.

(g) Find the value of n. [5]

7. 23N.1.SL.TZ1.8

The functions f and g are defined by

$f(x) = \ln(2x - 7)$, where $x > \frac{7}{2}$

$g(x) = 2\ln x - \ln d$, where $x > 0,\ d \in \mathbb{R}^+$.

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

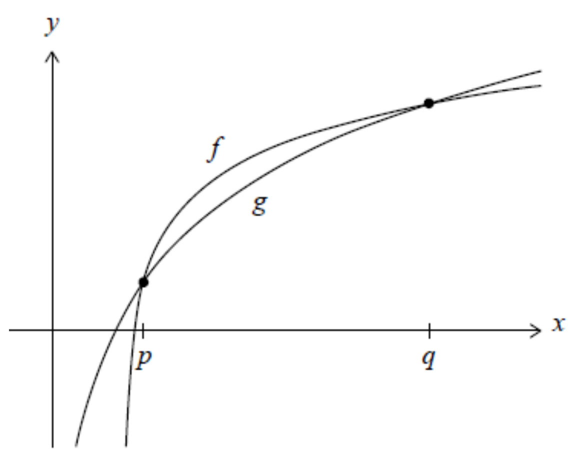
The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b.i) Show that, at the points of intersection, $x^2 - 2dx + 7d = 0$. [4]

(b.ii) Hence, show that $d^2 - 7d > 0$. [3]

(b.iii) Find the range of possible values of d. [2]

The following diagram shows parts of the graph $y = f(x)$ and $y = g(x)$.

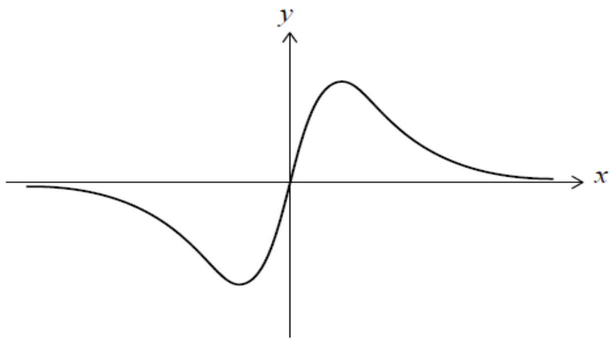


The graphs intersect at $x = p$ and $x = q$, where $p < q$.

(c) In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a,\ b \in \mathbb{Z}^+$. [5]

8. 23N.1.SL.TZ1.9

Consider the function f defined by $f(x) = \frac{12x}{(x^2+2)^3}$, where $x \in \mathbb{R}$. The graph of f is shown in the following diagram.



(a) Show that $f'(x) = \frac{12(2-5x^2)}{(x^2+2)^4}$. [4]

(b) Find $\int f(x)\,dx$. [4]

Consider a function $g(x)$ defined for $x \in \mathbb{R}$. The derivative of g is such that $g'(x) = f'(x)$, for all $x \in \mathbb{R}$.

Let R be e the region enclosed by the graph of f, the graph of g, the line $x = 0$ and the line $x = 3$. The area of R is $\frac{21}{2}$.

(c) Find the two possible expressions for $g(x)$. [5]

9. 23M.1.SL.TZ1.8

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

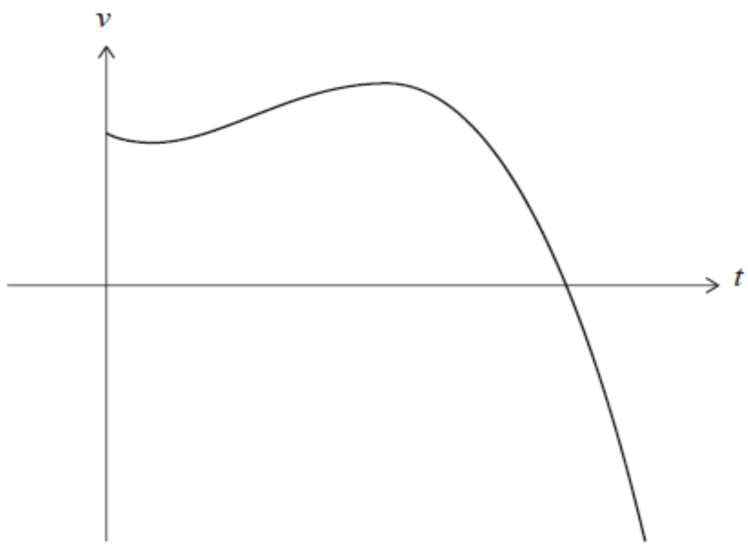
(d) Find the possible values of the common ratio, r. [3]

(e) Given that $v_{99} < 0$, find v_5. [2]

10. 23M.1.SL.TZ1.9

An object moves along a straight line. Its velocity, $v\,m\,s^{-1}$, at time t seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \leq t \leq 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$. [5]

(b) Find an expression for the acceleration of the object. [2]

(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

(d) Find the greatest speed reached by the object for $0 \leq t \leq 4$. [2]

(e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

11. 23M.1.SL.TZ2.8

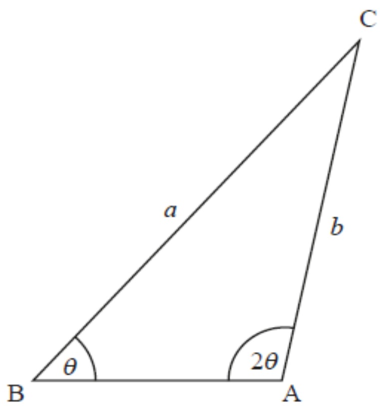
Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$.

(a) Find the value of

(a.i) $\sin\theta$; [2]

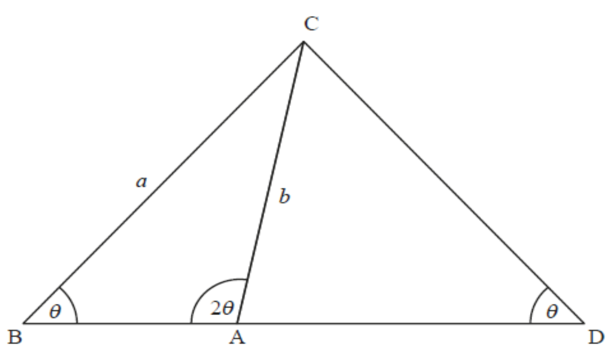
(a.ii) $\sin 2\theta$. [2]

The following diagram shows triangle ABC, with $\hat{B} = \theta$, $\hat{A} = 2\theta$, $BC = a$ and $AC = b$.



(b) Show that $b = \frac{3a}{4}$. [2]

$[BA]$ is extended to form an isosceles triangle DAC, with $\widehat{D} = \theta$, as shown in the following diagram.



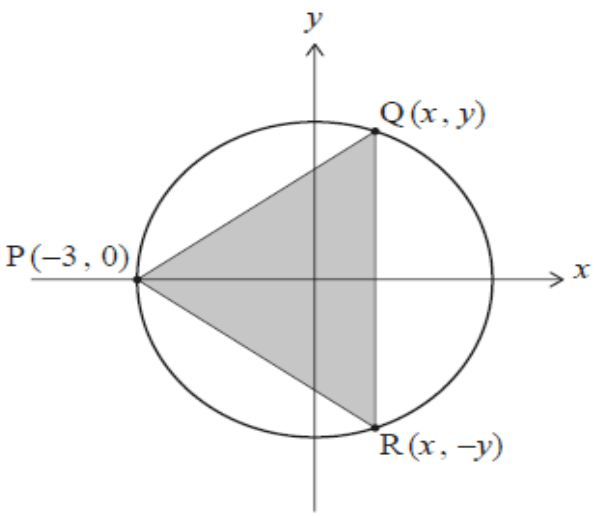
(c) Find the value of $\sin \mathrm{C}\hat{A}D$. [3]

(d) Find the area of triangle DAC, in terms of a. [5]

12. 23M.1.SL.TZ2.9

A circle with equation $x^2 + y^2 = 9$ has centre $(0,\ 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $P(-3,\ 0)$, $Q(x,\ y)$ and $R(x,\ -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



(a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]

(b) Hence, find an expression for A, the area of triangle PQR, in terms of x. [3]

(c) Show that $\frac{dA}{dx} = \frac{9-3x-2x^2}{\sqrt{9-x^2}}$. [4]

(d) Hence or otherwise, find the y-coordinate of R such that A is a maximum. [6]

13. 22M.1.SL.TZ1.8

Consider the series $\ln x + p\ln x + \frac{1}{3}\ln x + \cdots$, where $x \in \mathbb{R},\ x > 1$ and $p \in \mathbb{R},\ p \neq 0$.

Consider the case where the series is geometric.

(a.i) Show that $p = \pm\frac{1}{\sqrt{3}}$. [2]

(a.ii) Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. [3]

Now consider the case where the series is arithmetic with common difference d.

(b.i) Show that $p = \frac{2}{3}$. [3]

(b.ii) Write down d in the form $k\ln x$, where $k \in \mathbb{Q}$. [1]

(b.iii) The sum of the first n terms of the series is $-3\ln x$.

Find the value of n. [6]

14. 22M.1.SL.TZ1.9

(a.i) Expand and simplify $(1-a)^3$ in ascending powers of a. [2]

(a.ii) By using a suitable substitution for a, show that $1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x = 8\sin^6 x$. [4]

Consider $f(x) = 4\cos x(1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x)$.

(b.i) Show that $\int_0^m f(x)dx = \frac{32}{7}\sin^7 m$, where m is a positive real constant. [4]

(b.ii) It is given that $\int_m^{\frac{\pi}{2}} f(x)dx = \frac{127}{28}$, where $0 \leq m \leq \frac{\pi}{2}$. Find the value of m. [5]

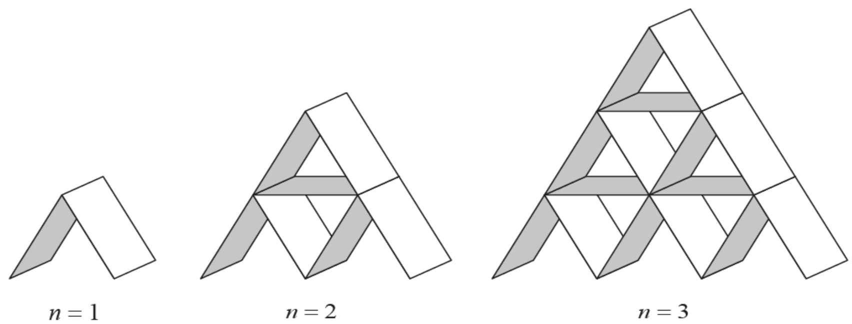
PAPER 2   With GDC

15. 25M.2.SL.TZ1.8

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of $60°$ to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

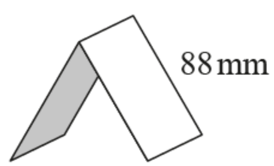
(c) Show that $t_n = \frac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

The long edge of each playing card measures $88\,mm$ as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

16. 25M.2.SL.TZ1.9

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R},\ x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $P\ \left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0,\ 0)$.

(b.i) Find the value of k. [3]

(b.ii) Hence, write down the coordinates of P. [1]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R},\ x \neq -\frac{d}{c}$, and $c,\ d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(b) In terms of c and d, write down the equation of

(c.i) the vertical asymptote; [1]

(c.ii) the horizontal asymptote. [1]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d. [2]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]

17. 25M.2.SL.TZ2.8

At Adam's Apple Orchard the weights of apples, W, in grams, are normally distributed with a mean 175 grams and standard deviation 8 grams.

(a) Find the probability that a randomly chosen apple weighs less than 170 grams.[2]

(b) It is found that 20 % of the apples weigh more than w grams. Find w, correct to four significant figures. [2]

All orchards classify an apple as premium when its weight is between 170 and 185 grams.

(c) Find the percentage of apples that are classified as premium at Adam's Apple Orchard.

[2]

After orders are completed, there are many apples left over. Boxes are filled with randomly chosen left-over apples. Each box contains 40 apples.

(d) Find the probability that a randomly chosen box contains at least 30 premium apples.

[3]

(e) If 10 of these boxes are randomly selected, find the probability that exactly 4 boxes have at least 30 premium apples. [2]

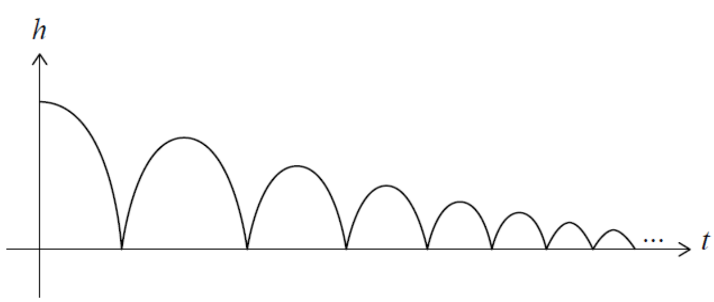
At a neighbouring orchard the weights of apples, M, in grams, are normally distributed with mean μ and standard deviation σ. It is known that:

- $82\,\%$ of their apples are classified as premium
- the percentage of apples that weigh less than 170 grams is twice the percentage of apples that weigh more than 185 grams.

(f) Find the value of μ. [6]

18. 25M.2.SL.TZ2.9

A tennis ball is dropped from a height. After each bounce the maximum height reached by the ball is $\frac{2}{3}$ of its previous maximum height. This can be seen in the diagram below where h, in metres, is the height of a ball after t seconds.



A box contains tennis balls. Each ball satisfies the condition of rebounding to $\frac{2}{3}$ of their previous maximum height. The tennis balls are numbered Ball $1,\ 2,\ 3.\ldots$

Ball 1 is dropped from a height of 10 metres.

(a) Find the maximum height of Ball 1 after the 5^{th} bounce. [3]

(b) Find the total distance travelled by Ball 1 immediately before the 5^{th} bounce. [3]

Let δ be the total distance travelled by any of these balls.

(c) A ball is dropped from a height of x metres. Show that $\delta = 5x$ metres. [3]

Let δ_1 be the total distance travelled by Ball 1.

(d) Write down the value of δ_1. [1]

Ball 2 is dropped from a height of 9.56 metres.

Let δ_2 be the total distance travelled by Ball 2, and so on for each ball in the box.

It is given that $\delta_1,\ \delta_2\ ,\ \delta_3\ ...$ form an arithmetic sequence.

(e) Determine which tennis ball is the first ball to travel less than 25 metres. [4]

19. 25M.2.SL.TZ3.8

Consider a discrete random variable X.

(a) State two conditions required for X to be modelled by a binomial distribution. [2]

A water theme park has two rides: *Daifong* and *Torbellino*. Each visitor's decision to ride on either *Daifong* or *Torbellino* is made independently of any other person.

From previous records, it is expected that 37 % of the visitors on any particular day will ride *Daifong*.

On Saturday, 1 900 people will visit the theme park.

(b) Find the number of people that are expected to ride *Daifong*. [2]

(c) Find the probability that

(c.i) 712 people will ride *Daifong*; [2]

(c.ii) between 684 and 712 people, inclusive, will ride *Daifong*. [2]

(d) Given that between 684 and 712 people, inclusive, will ride *Daifong*, find the probability that at most 692 people will ride *Daifong*. [4]

The ride *Torbellino* is more popular at the theme park. It is expected that $61\,\%$ of the visitors on any particular day will ride *Torbellino*.

It can be assumed that the probability a person will ride *Daifong* is independent of them riding *Torbellino*.

(e) Find the probability that a person will ride both *Daifong* and *Torbellino*. [2]

Next Tuesday n people will visit the theme park. The probability that at most 500 people will ride *Torbellino* is approximately 0.693.

(f) Find the value of n. [3]

20. 25M.2.SL.TZ3.9

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in $m\,s^{-1}$, during the race can be modelled by $v\,(t) = \frac{8.14\,t}{\sqrt{t^2+0.2}}$, where $t \geq 0$. Time, t, is measured in seconds from when the race starts.

(a.i) Write down the value of $v\,(1)$. [1]

(a.ii) Find the time when Fiona's velocity is $5\,m\,s^{-1}$. [2]

(b) Find the time when Fiona's acceleration is $4\,m\,s^{-2}$. [2]

(c.i) Write down the limit of $v\,(t)$ as t approaches infinity. [2]

(c.ii) State a reason why the value in part (c)(i) is not valid in the context of this question [1]

Lucy's velocity, in $m\,s^{-1}$, during the race can be modelled by $w\,(t) = \frac{8\,t}{\sqrt{t^2+0.3}}$, where $t \geq 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

(d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres.

[6]

21. 24N.2.SL.TZ1.8

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.6	32.3	34.1	35.6	38.0

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

(a.i) Write down the value of a and the value of b. [2]

(a.ii) Interpret, in context, the value of a. [1]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.4 million people.

(b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 30.6(1.007)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

(c) (c.i)

Use Benoit's model to predict the population of Canada in the year 2100. [2]

(c.ii) Interpret, in context, the value 1.007 in Benoit's model. [1]

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{61}{1+e^{-0.03t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

(d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]

(e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]

(f) Find the value of

(f.i) $B'(40)$; [1]

(f.ii) $C'(40)$. [1]

(g) Compare and interpret, in context, the values of $B'(40)$ and $C'(40)$. [2]

22. 24N.2.SL.TZ1.9

In this question all values of x and t are in radians.

Consider the function $f(x) = 3\sin(4\pi x)$.

(a.i) Write down the amplitude of the graph of f. [1]

(a.ii) Find the period of f. [2]

Consider a second function $g(x) = -4\cos(4\pi x)$.

The sum of these functions can be expressed in the form $f(x) + g(x) = a\cos\big(b(x-c)\big)$, where $a,\ b,\ c > 0$.

(b) By considering the graph of $y = f(x) + g(x)$, determine

(b.i) the value of a; [2]

(b.ii) the value of b; [1]

(b.iii) the smallest possible value of c. [1]

A car is travelling along a straight residential street with speed bumps placed at regular intervals on the road to encourage safer driving. The car travels at a minimum velocity when passing over speed bumps and reaches a maximum velocity in between speed bumps.

Its velocity, in $m\,s^{-1}$, can be modelled by the function $v(t) = -3.5\cos\left(\frac{\pi}{14}(t-5)\right) + 9$, where t is measured in seconds.

(c) Find the time at which the car first reaches its maximum velocity. [1]

(d) Find the number of speed bumps the car passes over in the first two minutes of motion.

[1]

(e.i) Find $v'(t)$. [2]

(e.ii) Hence, or otherwise, write down the maximum acceleration of the car. [2]

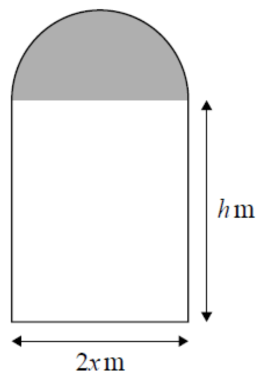
(f) Find the distance, in metres, between consecutive speed bumps. [3]

23. 24M.2.SL.TZ2.8

A window is designed in the shape of a semicircle attached to a rectangle.

The rectangular section of the window has dimensions $2x$ metres by h metres.

The window consisting of its two sections is shown in the following diagram.



Let the area of the window be A square metres.

(a) Write down an expression for A in terms of x and h. [2]

Let the perimeter of the window be P metres.

(b) Given that $P = 10$, show that $h = \frac{1}{2}(10 - 2x - \pi x)$. [2]

The window is designed to let in the maximum amount of light.

The rectangular section of the window consists of clear glass and lets in three units of light per square metre.

The semicircular section of the window consists of tinted glass and lets in one unit of light per square metre.

(c) Show that the amount of light, L units, let in by the window is given by $L = 30x - 6x^2 - \frac{5}{2}\pi x^2$. [4]

(d.i) Find an expression for $\frac{dL}{dx}$. [2]

(d.ii) Find the value of x so that the window lets in the maximum amount of light. Justify that this value of x gives a maximum. [3]

(d.iii) Find the value of h so that the window lets in the maximum amount of light. [2]

24. 24M.2.SL.TZ2.9

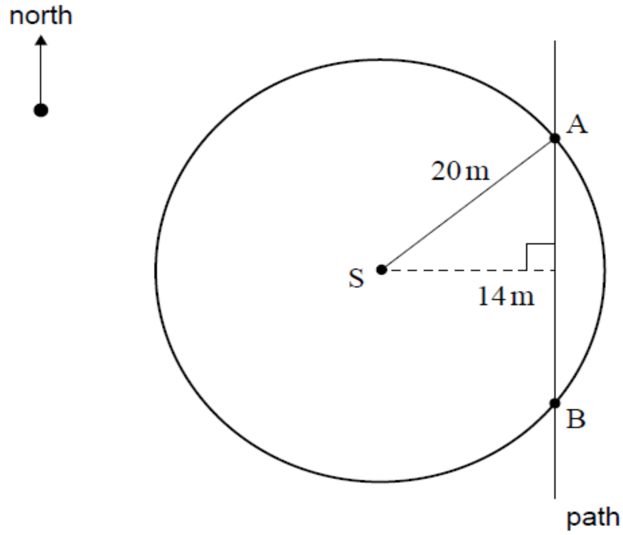
A rotating sprinkler is at a fixed point S.

It waters all points inside and on a circle of radius 20 metres.

Point S is 14 metres from the edge of a path which runs in a north-south direction.

The edge of the path intersects the circle at points A and B.

This information is shown in the following diagram.



(a) Show that $AB = 28.57$, correct to four significant figures. [3]

The sprinkler rotates at a constant rate of one revolution every 16 seconds.

(b) Show that the sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second. [1]

Let T seconds be the time that $[AB]$ is watered in each revolution.

(c) Find the value of T. [4]

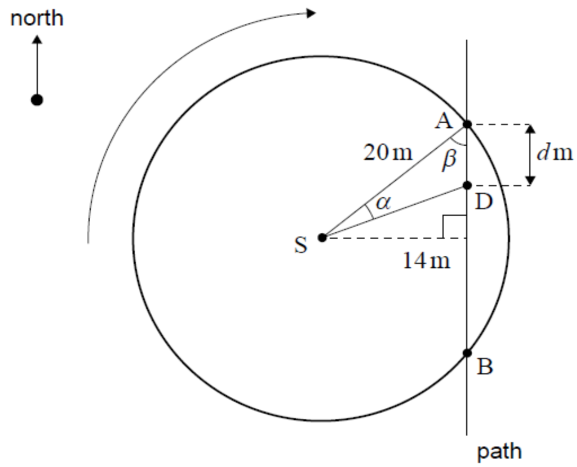
Consider one clockwise revolution of the sprinkler.

At $t = 0$, the water crosses the edge of the path at A.

At time t seconds, the water crosses the edge of the path at a movable point D which is a distance d metres south of point A.

Let $\alpha = A\hat{S}D$ and $\beta = S\hat{A}B$, where $\alpha,\ \beta$ are measured in radians.

This information is shown in the following diagram.



(d) Write down an expression for α in terms of t. [1]

It is known that $\beta = 0.7754$ radians, correct to four significant figures.

(d) By using the sine rule in ΔASD, show that the distance, d, at time t, can be modelled by

$$d(t) = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}.$$ [3]

A turtle walks south along the edge of the path.

At time t seconds, the turtle's distance, g metres south of A, can be modelled by

$g(t) = 0.05t^2 + 1.1t + 18$, where $t \geq 0$.

(f) At $t = 0$, state how far south the turtle is from A. [1]

Let w represent the distance between the turtle and point D at time t seconds.

(g.i) Use the expressions for $g(t)$ and $d(t)$ to write down an expression for w in terms of t.

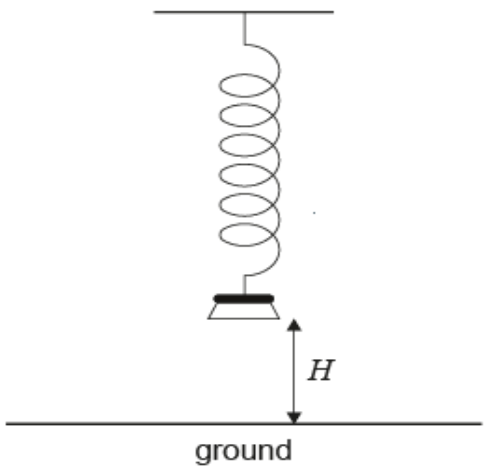
[1]

(g.ii) Hence find when and where on the path the water first reaches the turtle. [3]

25. 23M.2.SL.TZ2.8

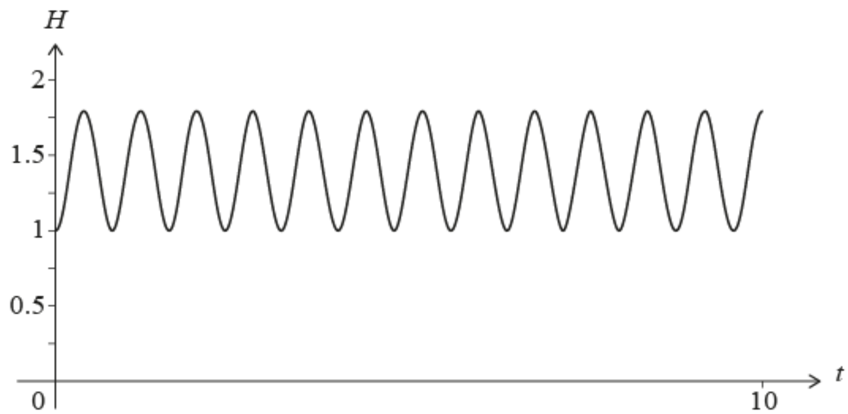
A weight suspended on a spring is pulled down and released, so that it moves up and down vertically.

The height, H metres, of the base of the weight above the ground can be modelled by the function $H(t) = a\cos(7.8t) + b$, for $a, b \in \mathbb{R}$ and $0 \leq t \leq 10$, where t is the time in seconds after the weight is released.



(a) Find the period of the function. [2]

The weight is released when its base is at a minimum height of 1 metre above the ground, and it reaches a maximum height of 1.8 metres above the ground. The graph of H is shown in the following diagram.



(b) Find the value of

(b.i) a; [2]

(b.ii) b. [1]

(c) Find the number of times that the weight reaches its maximum height in the first five seconds of its motion. [2]

(d) Find the first time that the base of the weight reaches a height of 1.5 metres. [2]

A camera is set to take a picture of the weight at a random time during the first five seconds of its motion.

(e) Find the probability that the height of the base of the weight is greater than 1.5 metres at the time the picture is taken [4]

26. 23M.2.SL.TZ2.9

A bag contains n balls. It is known that ten of the balls are green, and the rest of the balls are red. Balls are drawn from the bag, one after the other, without replacement.

(a) Find, in terms of n, the probability that

(a.i) the first ball drawn is green; [1]

(a.ii) the first two balls are green. [2]

For the following parts of this question, let $n = 25$.

(b) Show that the probability that the first two balls are red is 0.35. [2]

(c) Find the probability that the first three balls are all red. [2]

(d) Find the probability that at least one of the first three balls is green. [2]

A game is played where **four** balls are drawn, one after the other, from the bag of 25 balls, without replacement. A player earns points based on when the first green ball is drawn. At the end of each game, the four balls are put back in the bag.

A player earns zero points if no green ball is picked, or if the first green ball is picked on the first or second draw.

A player earns 10 points if the first green ball is picked on the third draw and earns 50 points if the first green ball is picked on the fourth draw.

Millie plays this game k times. She finds her score by adding together her points from each game.

(e) Find the least value of k such that Millie's expected score is greater than 100. [6]

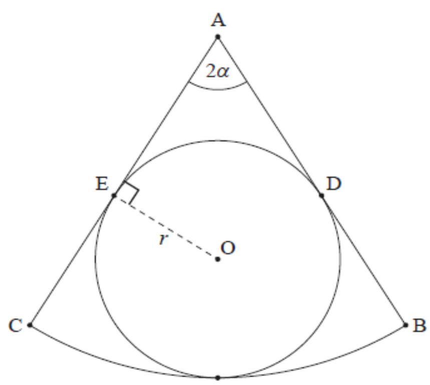
27. 22N.2.SL.TZ0.8

The following diagram shows a sector ABC of a circle with centre A. The angle $\mathrm{B\hat{A}C} = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $\mathrm{O\hat{E}A} = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

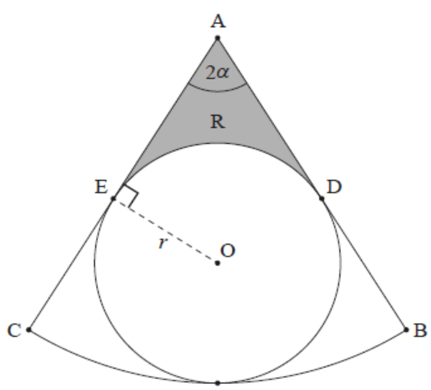
diagram not to scale



(a) Show that the area of the quadrilateral ADOE is $\frac{r^2}{\tan\alpha}$. [4]

R represents the shaded region shown in the following diagram.

diagram not to scale



(b.i) Find $\mathrm{D\hat{O}E}$ in terms of α. [2]

(b.ii) Hence or otherwise, find an expression for the area of R. [3]

(c) Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r. [4]

28. 22N.2.SL.TZ0.9

The time worked, T, in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

(a) Find the probability that an employee selected at random works more than 40 hours per week. [2]

(b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week. [3]

A large group of employees work more than 40 hours per week.

(c.i) An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week. [4]

(c.ii) Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week. [3]

It is known that $\text{P}(a \leq T \leq b) = 0.904$ and that $\text{P}(T > b) = 2\text{P}(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

(d Find the maximum time, in hours per week, that an employee can work and still be considered part-time.

[4]

29. 22M.2.SL.TZ1.8

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R},\ x \neq -4$.

For the graph of f

(a.i) write down the equation of the vertical asymptote. [1]

(a.ii) find the equation of the horizontal asymptote. [2]

(b.i) Find $f^{-1}(x)$. [4]

(b.ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. [4]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

(c.i) Find the value of p and the value of q. [2]

(c.ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1}. [3]

30. 22M.2.SL.TZ1.9

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

(a) Find the probability that a randomly selected chocolate muffin weighs less than 61 g. [2]

(b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. [2]

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

(c.i) Find the probability that the randomly selected muffin weighs less than 61 g. [4]

(c.ii) Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. [3]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

(d) Find the value of σ. [5]