Q1.

(a) (i) $A=\frac{1}{2}\times 6\times q+\frac{1}{2}\times 8\times p+48$ **OR** $A=\frac{1}{2}(p+6)(q+8)$ **OR**
$A=3q+4p+48$ ***A1***

(ii) valid attempt to link p and q, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta=\frac{8}{p}$ and $\tan\theta=\frac{q}{6}$ **OR** $\tan\theta=\frac{p}{8}$ and $\tan\theta=\frac{6}{q}$ **OR** $\frac{8}{p}=\frac{q}{6}$

correct equation linking p and q ***A1***

eg. $pq=48$ **OR** $p=\frac{48}{q}$ **OR** $q=\frac{48}{p}$

substitute $p=\frac{48}{q}$ into a correct area expression ***M1***

eg. $(A=)\frac{1}{2}\times 6\times q+\frac{1}{2}\times 8\times\frac{48}{q}+48$ **OR** $(A=)\frac{1}{2}\left(\frac{48}{q}+6\right)(q+8)$

$A=3q+\frac{192}{q}+48$ ***AG***

Note: The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2}+3$ ***A1A1***

Note: Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for 3. Award ***A1A0*** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2}+3=0$ ***A1***

(ii) $q=8$ cm ***A1***

[2 marks]

Total [8 marks]

Q2.

(a) recognition of need to integrate (*eg* reverse power rule or integral symbol) ***(M1)***

$P(x) = -0.8x^2 + 48x\ (+c)$ ***A1A1***

$260 = -0.8 \times (15)^2 + 48 \times (15) + c$ ***(M1)***

Note: Award ***M1*** for correct substitution of $x = 15$ and $P = 260$. A constant of integration must be seen (can be implied by a correct answer).

$c = -280$

$P(x) = -0.8x^2 + 48x - 280$ ***A1***

[5 marks]

(b) profit will decrease (with each new car produced) ***A1***

EITHER

because the profit function is decreasing / the gradient is negative / the rate of change of P is negative ***R1***

OR

$\int_{30}^{50} -1.6x + 48\ (\mathrm{d}x) = -320$ ***R1***

OR

evidence of finding $P(30) = 440$ and $P(50) = 120$ ***R1***

Note: Award at most ***R1A0*** if $P(30)$ or $P(50)$ or both have incorrect values.

[2 marks]

Total [7 marks]

Q3.

(a) 3 **A1**

Note: Accept (3, 0) seen.

[1 mark]

(b) **METHOD 1**

$0 = 4a - 2b + c$, $0 = 9a + 3b + c$, $-\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$ **(M1)(A1)**

(i) 2 **A1**

(ii) -2 **A1**

(iii) -12 **A1**

Note: Award the ***(M1)(A1)*** if at least one correct value is seen.
Do not apply ***FT*** form part (a) if workings are not shown.

METHOD 2

$-12.5 = a(0.5 + 2)(0.5 - 3)$ **(M1)**

(i) $a = 2$ **A1**

$0 = 2 \times (3)^2 + 3b + c$
$0 = 2 \times (-2)^2 + (-2)b + c$ **(M1)**

(ii) $b = -2$ **A1**

(iii) $c = -12$ **A1**

[5 marks]

(c) $x = 0.5$ **A1**

Note: Do not ***FT*** from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

Q4.

(a) use of geometric sequence with $r = 0.85$ **M1**

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ **A1**
$= 0.68$ m
$= 68$ cm **AG**

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ **A1**
$= 68$ cm **AG**

[2 marks]

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If 1.8 m (or 180 cm) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If 1.53 m (or 153 cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114$ m and $(0.85)^{18}(1.8) = 0.0966$ m ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \dfrac{(1.8 \times 0.85)(1-0.85^3)}{1-0.85}$ $(= 3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(3.935925)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \dfrac{(1.8)(1-0.85^4)}{1-0.85}$ $(= 5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8)$ $(= 3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

Note: Answers may be given in cm.

[3 marks]
[Total 7 marks]

Q5.

(a) **METHOD 1**

(when $t = 2$)

$\frac{dP}{dt} = -4$ **OR** $\frac{dP}{dt} < 0$ (equivalent in words) **OR** $3(2)^2 - 8(2) = -4$ ***M1***

therefore P is decreasing ***A1***

METHOD 2

sketch with $t = 2$ indicated in 4th quadrant **OR** t-intercepts identified ***M1***

therefore P is decreasing ***A1***

[2 marks]

(b) $(P(t) =)\ t^3 - 4t^2\ (+c)$ ***A1A1***

$4 = 1^3 - 4(1)^2 + c$ ***(M1)***

Note: Award ***M1*** for substituting $(1, 4)$ into their equation with $+c$ seen.

$c = 7$

$P(t) = t^3 - 4t^2 + 7$ ***A1***

[4 marks]

[Total 6 marks]

Q6.

(a) $\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

(b) **EITHER**

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000\ (= 10^5)$ ***A1***

[2 marks]

(c) $0.001 < N < 100000\ (10^{-3} < N < 10^5)$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for correct inequalities/interval notation.

[2 marks]

(d) $N = \frac{10^5}{10^{7.2}}\ (= 0.0063095...)$ ***(M1)***

length of time = $\frac{1}{0.0063095...} = 10^{2.2}$

= 158 years ***A1***

[2 marks]

[Total 8 marks]

Q7.

(a) (let T be the number of passengers who arrive)

$(\mathrm{P}(T > 72) =)\ \mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74,\ 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379\ \ (0.00379124...)$ ***A1***

Note: Using the distribution $\mathrm{B}(74,\ 0.1)$, to work with the 10% that do not arrive for the flight, here and throughout this question, is a valid approach.

[3 marks]

(b) (i) 72×0.9 ***(M1)***

64.8 ***A1***

(ii) $n \times 0.9 = 72$ ***(M1)***

80 ***A1***

[4 marks]

(c) **METHOD 1**

EITHER

when selling 74 tickets

	$T \leq 72$	$T = 73$	$T = 74$
Income minus compensation (I)	11100	10800	10500
Probability	0.9962…	0.003380…	0.0004110…

top row ***A1A1***

bottom row ***A1A1***

Note: Award ***A1A1*** for each row correct. Award ***A1*** for one correct entry and ***A1*** for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962... + 10800 \times 0.00338... + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

OR

income is $74 \times 150 = 11100$ ***(A1)***

expected compensation is

$0.003380... \times 300 + 0.0004110... \times 600\ \ (= 1.26070...)$ ***(M1)A1A1***

expected income when selling 74 tickets is $11100 - 1.26070\ldots$ ***(M1)***

$= 11098.73..\ \ (= \$11099)$ ***A1***

THEN

income for 72 tickets $= 72 \times 150 = 10800$ ***(A1)***

so expected gain $\approx 11099 - 10800 = \$299$ ***A1***

METHOD 2

for 74 tickets sold, let C be the compensation paid out

$\mathrm{P}(T=73)=0.00338014\ldots,\ \mathrm{P}(T=74)=0.000411098\ldots$ ***A1A1***

$\mathrm{E}(C)=0.003380...\times 300+0.0004110...\times 600\ \ (=1.26070...)$ ***(M1)A1A1***

extra expected revenue $=300-1.01404...-0.246658\ldots\ \ (300-1.26070\ldots)$

(A1)(M1)

Note: Award ***A1*** for the 300 and ***M1*** for the subtraction.

$=\$299$ (to the nearest dollar) ***A1***

METHOD 3

let D be the change in income when selling 74 tickets.

	$T\le 72$	$T=73$	$T=74$
Change in income	300	0	−300

(A1)(A1)

Note: Award ***A1*** for one error, however award ***A1A1*** if there is no explicit mention that $T=73$ would result in $D=0$ and the other two are correct.

$\mathrm{P}(T\le 73)=0.9962...,\ \mathrm{P}(T=74)=0.000411098\ldots$ ***A1A1***

$\mathrm{E}(D)=300\times 0.9962...+0\times 0.003380...-300\times 0.0004110$ ***(M1)A1A1***

$=\$299$ ***A1***

[8 marks]

[Total 15 marks]

Q8.

(a) **METHOD 1** – *(with FV = 4000)*

EITHER

N= 10
I=1.5
FV= 4000
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for $(3.5-2=)\ 1.5$ seen and ***M1*** for all other entries correct.

OR

$4000=A(1+0.015)^{10}$ ***(A1)(M1)***

Note: Award ***A1*** for 1.5 or 0.015 seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN

(PV =) $3447 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 2 – *(With FV including inflation)*
calculate FV with inflation
4000×1.02^{10} ***(A1)***
(=4875.977…)

EITHER
$4000\times1.02^{10} = \text{PV}\times1.035^{10}$ ***(A1)***
OR
N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for *their* FV and all other entries correct.

THEN
(PV =) $3457 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 3 – *(Using formula to calculate real rate of return)*
(real rate of return =) 1.47058…(%) ***(A1)***

EITHER
$4000 = \text{PV}\times1.0147058\ldots^{10}$ ***(A1)***
OR
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for all entries correct.

THEN
(PV =) $3457 ***A1***
[3 marks]

(b) **METHOD 1** – *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) $4861.87 **OR** $4875.97 ***(A1)***
so payment required (from TVM) will be $294 **OR** $295 ***A1***

Note: Award ***A0*** if a negative sign given, unless already penalized in part (a).

METHOD 2 – *(Using FV)*
N= 10
I=3.5
PV= –1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for I=3.5 **and** FV= ±4875.977…, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) $295 (295.393) ***A1***

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

[3 marks]
Total [6 marks]

Q9.

(a) $\left(f'(x)=\right)\ 2x+\frac{3}{x^2}$ ***A1A1***

Note: Award ***A1*** for $2x$, ***A1*** for $+\frac{3}{x^2}$ **OR** $+3x^{-2}$.

[2 marks]

(b) attempt to substitute 1 into their part (a) ***(M1)***

$\left(f'(1)=\right)\ 2(1)+\frac{3}{1^2}$

5 ***A1***

[2 marks]

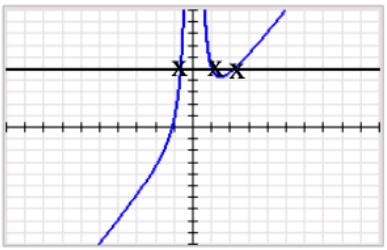
(c) **EITHER**

$5=2x+\frac{3}{x^2}$ ***M1***

$x=-0.686,\ 1,\ 2.19\ \ (-0.686140...,\ 1,\ 2.18614\ldots)$ ***A1***

OR

sketch of $y=f'(x)$ with line $y=5$ ***M1***



three points of intersection marked on this graph ***A1***
(and it can be assumed no further intersections occur outside of this window)

THEN

there are two other tangent lines to $f(x)$ that are parallel to L ***A1***

Note: The final ***A1*** can be awarded provided two solutions other than $x=1$ are shown **OR** three points of intersection are marked on the graph.

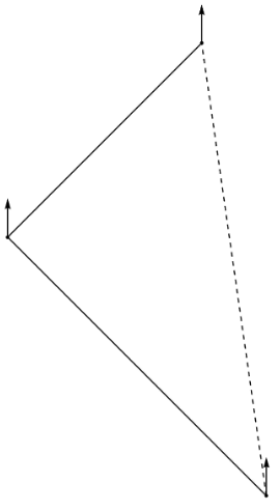
Award ***M1A1A1*** for an answer of "3 lines" where L is considered to be parallel with itself (given guide definition of parallel lines), but only if working is shown.

[3 marks]

Total [7 marks]

Q10.

diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



Note: Values do not need to be seen on the diagram to award the ***A1***.

recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

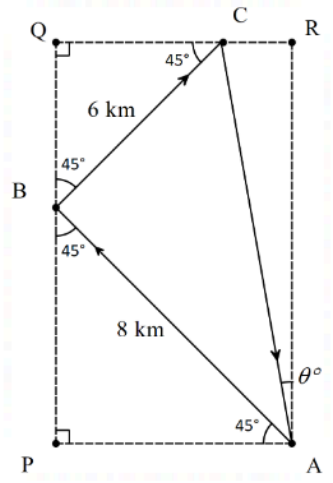
e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

$=172^\circ$ $(171.869...^\circ)$ ***A1***

METHOD 2

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$

(these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked AP, BP, CQ and BQ in the diagram ***(M1)***

$$AP = BP = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$CQ = BQ = 6\frac{\sqrt{2}}{2} = 4.2426...$$

Note: This may be done using a vector approach.

using $\tan\theta° = \frac{AP - CQ}{PB + BQ}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$= 172° \quad (171.869...°)$ ***A1***

[Total: 5 marks]

Q11.

(a) attempt to find the difference between 75.7 and 67.3 ***(M1)***

$\frac{75.7-67.3}{2}$

$4.2\ (\text{km h}^{-1})$ ***A1***

[2 marks]

(b) recognition of normal distribution that includes 72 ***(M1)***

e.g., sketch of normal distribution curve with 72 labelled to the right of the mean **OR**
Normal CDF calculation using 72

$0.132\quad (0.131559\ldots,\ 13.2\%,\ 13.1559\ldots\%)$ ***A1***

[2 marks]

(c) **METHOD 1 (Comparing areas above and below the mean)**

$\text{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF$(67.3,\ 74,\ 67.3,\ 4.2)$ **OR** sketch of normal distribution with 67.3 and 74 labelled and shaded between ***(M1)***

area of region between mean and q is at least $0.445\ \ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ \ (0.375329\ldots)$ between mean and p ***R1***

The region between p and q is not symmetrical ***AG***

METHOD 2 (Comparing areas in the tails)

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ ***(M1)***

$\text{P}(\text{speed} < 74) = 0.944670\ldots$

$\text{P}(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ 0.124670\ldots$

$\text{P}(\text{speed} > q) = (1 - 0.944670\ldots =)\ \ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $\text{P}(\text{speed} > q) \leq 0.0553295$ and $\text{P}(\text{speed} < p) \geq 0.124670$ so

$\text{P}(\text{speed} > q)$ will never equal $\text{P}(\text{speed} < p)$ ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 3 (Assumption of symmetry comparing speeds)

attempt to calculate area below q assuming distribution is symmetrical ***(M1)***

e.g. $\mathrm{P}(\text{speed}<q)=0.82+\frac{1}{2}\times 0.18 \quad (0.91)$

EITHER

$(q=)\ 72.9 \quad (72.9311\ldots)$ ***A1***

$72.9<74$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$\mathrm{P}(\text{speed}<74)=0.945 \quad (0.944670\ldots)$ ***A1***

$0.945>0.91$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 4 (Assumption of symmetry comparing areas)

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6<\text{speed}<74)$ **OR** Normal CDF(60.6, 74, 67.3, 4.2) **OR**

$\mathrm{P}(67.3<\text{speed}<74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2)

EITHER

$0.889 \quad (0.889340\ldots)$ ***A1***

$0.889>0.82$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$0.445 \quad (0.444670\ldots)$ ***A1***

$0.445>0.82\div 2$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

[3 marks]

[Total: 7 marks]

Q12.

(a) 14.55 (cm) to 14.65 (cm) ***A1A1***

Note: Award ***A1*** for each value. Accept $14.55 \leq AC < 14.65$.

[2 marks]

(b) attempt to use Pythagorean theorem **OR** trig ratio to find slant height ***(M1)***

a correct expression for either the **upper** or **lower** bound ***(A1)***

$\sqrt{14.55^2 - 10^2}$ **OR** $\sqrt{14.65^2 - 10^2}$ **OR**

$\sin(46.5844...^\circ) = \frac{AH}{14.55}$ **OR** $\sin(46.9533...^\circ) = \frac{AH}{14.65}$

(lower bound =) 10.6 (cm) (10.5689…) **AND**

(upper bound =) 10.7 (cm) (10.7061…) ***A1***

[3 marks]

(c) **METHOD 1**

attempt to find the maximum angle measure of the post using trigonometry ***(M1)***

e.g. $\cos\theta = \frac{10}{10.7061...}$ **OR** $\frac{\sin\theta}{3.82393\ldots} = \frac{\sin(90^\circ)}{10.7061\ldots}$

Note: Accept an inequality.

$(\theta =)\ 20.9\ (^\circ)$ $(20.9265\ldots(^\circ))$ ***A1***

and hence the post is safe ***AG***

Note: Use of radians gives an answer of 0.365 $(0.365237\ldots)$; award at most ***(M1)A0*** since this value cannot be directly compared to 22°.

Award at most ***(M1)A0*** for an angle calculated using their lower bound from part (b).

METHOD 2

attempt to find the longest slant height for angle to be a maximum of 22° ***(M1)***

e.g. $\cos(22^\circ) = \frac{10}{x}$

$(x = 10.7853\ldots)$

$10.7061\ldots < 10.7853\ldots$ ***A1***

and hence the post is safe ***AG***

Note: A comparison to their upper bound from part (b) is required for ***A1*** to be awarded. Use of radians gives an unreasonable answer of $-10.0003...$; award at most ***(M1)A0***.

[2 marks]

Total [7 marks]

Q13.

(a) attempt at using the trapezoidal rule ***(M1)***

$\text{area} = \frac{1}{2}\left(3 + 2(8+19) + 42\right)$

$= 49.5\ (\text{m}^2)$ ***A1***

[2 marks]

(b) recognition of need to integrate (*e.g.* reverse power rule or integral symbol) ***(M1)***

$\int 3x^2 + 4\,\text{d}x = x^3 + 4x + c$ ***(A1)(A1)***

Note: Award ***A1*** for each correct term.

$f(x) = x^3 + 4x + 3$ ***A1***

Note: Award ***A1*** for simplified correct answer including the value of c. Accept a value of c of 3.005 or 3.025 or 2.975 for using the non-integer x-values and their corresponding y-values.

[4 marks]

(c) **METHOD 1**

forming expression for sum of integral and deconstructing the trapezoid into a rectangle and triangle ***(M1)***

$\int_0^3 x^3 + 4x + 3\ \text{d}x (= 47.25) + 42 \times 1 + \frac{1}{2} \times 2 \times 42\ (= 84)$ ***(A1)***

$= 131\ (\text{m}^2)\ \ (131.25)$ ***A1***

METHOD 2

forming expression for sum of integral and trapezoid ***(M1)***

$\int_0^3 x^3 + 4x + 3\ \text{d}x (= 47.25)\ + \frac{1}{2} \times 4 \times 42\ (= 84)$ ***(A1)***

$= 131\ (\text{m}^2)\ \ (131.25)$ ***A1***

Note: Award ***(A1)*** for their integral with the correct limits added to 84 or their 47.25 added to 84.

[3 marks]

Total [9 marks]

Q14.

(a) $0.15 + 0.2 + k + 0.16 + 2k + 0.25 = 1$ ***(M1)***

$k = 0.08$ ***A1***

[2 marks]

(b) $(-4 \times 0.15) + (-3 \times 0.2) + (-1 \times 0.08) + (0 \times 0.16) + (1 \times 0.16) + (4 \times 0.25)$ ***(M1)***

$= -0.12$ ***A1***

$\text{E}(X) \neq 0$ therefore the game is not fair ***R1***

Note: Do not award ***A0R1*** without an explicit value for $\text{E}(X)$ seen. The ***R1*** can be awarded for comparing **their** $\text{E}(X)$ to zero provided working is shown.

[3 marks]

Total [5 marks]

Q15. (a)

Markscheme
78 ***A1***
[1 mark]

Markscheme
65 ***A1***
[1 mark]

Markscheme
EITHER (period =) 16 (could be seen on sketch) ***(M1)*** $b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$ $(b =)\ 0.393\ \left(0.392699\ldots, \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1*** **OR** $143 = 65\sin(4b) + 78$ ***(M1)*** $(\sin(4b) = 1)$ $(4b = \frac{\pi}{2}$ **OR** $4b = 90^\circ)$ $(b =)\ 0.393\ \left(0.392699\ldots, \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1*** ***[2 marks]***

Markscheme
13 ***A1***
Note: Apply follow through marking only if their final answer is positive.
[1 mark]

Markscheme
$(b =)\ 0.196\ \left(0.196349\ldots, \frac{\pi}{16}\right)$ **OR** $(b =) 11.3^\circ\ (11.25^\circ)$ ***A1***
[1 mark]

Q16.

(a) evidence of using binomial distribution **(M1)**

Note: Evidence is $X \sim \mathrm{B}(5,\ 0.72)$ or binomial with $n = 5,\ p = 0.72$.

$0.293 \quad (0.292626\ldots)$ **A1**

[2 marks]

(b) attempt to find the probability of taking a bus, (or *not* taking a bus); **(M1)**
$\mathrm{P(take\ bus)} = 0.28 \times 0.42,\ \mathrm{P(not\ take\ bus)} = 0.72 + 0.28 \times 0.58$

0.1176 or 0.8824 seen **(A1)**

EITHER
correct use of binomial distribution with their probability
$X \sim \mathrm{B}(5,\ 0.1176),\ X = 0$ **OR** $X \sim \mathrm{B}(5,\ 0.8824),\ X = 5$ **(A1)**
OR
$(1 - 0.1176)^5$ **OR** $(0.8824)^5$ seen **(A1)**

THEN
$0.535 \quad (0.534967\ldots)$ **A1**

[4 marks]
[Total 6 marks]

Q17.

(a) equating a volume of a half cylinder (or cylinder) to 0.8 **(M1)**

$0.8 = \frac{1}{2}\pi r^2 l$

$l = \frac{1.6}{\pi r^2}$ **A1**

Note: Do not accept decimal approximation of π for the ***A1*** given the demand of question.
Condone the use of h for l for the ***M1***

[2 marks]

(b) calculating area in terms of r and l ***M1***

$C = 2lr + \pi r^2 + \pi rl$

area with l replaced by $\frac{1.6}{\pi r^2}$ ***M1***

apply costs to correct part of each surface ***M1***

a correct substitution into an expression for C, leading to given answer

A1

e.g. $(C =)\ 4.40 \times \pi r\left(\frac{1.6}{\pi r^2}\right) + 4.40 \times 2r\left(\frac{1.6}{\pi r^2}\right) + p \times \pi r^2$

$(C =)\ 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$ ***AG***

Note: The ***AG*** line must be seen to award the final ***A1***.
No incorrect working should be seen after the correct substitution

[4 marks]

(c) **EITHER**

$\left(\frac{dC}{dr} =\right) -7.04r^{-2} - \frac{14.08}{\pi}r^{-2} + 2p\pi r$ ***A1A1A1***

OR

$-7.04r^{-2} - 4.48r^{-2} + 6.28pr \quad \left(-7.04r^{-2} - (4.48180...)r^{-2} + 6.28318...pr\right)$

A1A1A1

OR

$-11.5r^{-2} + 6.28pr \quad \left((-11.5218...)r^{-2} + 6.28318...pr\right)$ ***A2A1***

Note: Award ***A1*** for each correct term.
Award at most ***A1A1A0*** if extra terms are seen.

[3 marks]

(d) recognition of setting $\frac{dC}{dr}$ to zero ***(M1)***

attempt to substitute 0.7 in for r in their derivative ***(M1)***

$0 = -7.04(0.7)^{-2} - \frac{14.08}{\pi} \times (0.7)^{-2} + 2p\pi \times 0.7$

$(p =)$ (\$)5.35 (per square metre) ((\$) 5.34621…) ***A1***

Note: Accept \$5.34, as this will also lead to a radius of 0.7 (to 3sf).

[3 marks]

(e) attempt to calculate the cost of one container ***(M1)***

$(C =)\ 7.04(0.7)^{-1} + \frac{14.08}{\pi}(0.7)^{-1} + 5.34621...\pi \times 0.7^2$ ***(A1)***

Note: May be shown within a calculation of the cost of all containers.

$(C =)\ 24.6895…$

$24.6895… \times 350$

= (\$) 8641 ***A1***

Note: Answer must be rounded to the nearest dollar to award the final ***A1.***
Accept answers between 8641 and 8645 (inclusive), due to rounding the value of p and/or the cost of one container to the nearest cent.
Award ***(M1)(A1)A0*** for an answer rounded to 3sf (e.g. (\$)8640) or to 2dp (e.g., (\$)8641.35).
Accept an answer of (\$)8638 from use of \$5.34 in their cost calculation.

[3 marks]

(f) attempt to apply a discount of 8% to their part (e) ***(M1)***

Note: the discount percentage will depend on their answer to part (e)
e.g. $8641.35...\times 0.92$ **OR** $8641.35...\times 0.08$

($)7950 (($)7950.04…) ***A1***

[2 marks]

[Total: 17 marks]

Q18.

(a) (i) $x-3$ ***A1***

(ii) attempt to use 1200 to find width of park in terms of only x ***(M1)***
$\frac{1200}{x}$ (seen) **OR** $1200 = x \times$ park width **OR** $1200 = x \times$ (garden width $+4$)

$\frac{1200}{x}-4$ ***A1***

(iii) $A=(x-3)\times\left(\frac{1200}{x}-4\right)$ ***A1***

$=1200-4x-\frac{3600}{x}+12$ ***A1***

Note: Award first ***A1FT*** for multiplying *their* garden length and width and second ***A1*** for a simplified (parentheses removed) expression for A that leads to the given answer. The given answer must be shown for the second ***A1*** mark to be awarded

$=1212-4x-\frac{3600}{x}$ ***AG***

[5 marks]

(b) setting $1212-4x-\frac{3600}{x}=800$ (accept a sketch) ***(M1)***

$x=9.64\ (9.64011\ldots)\ (\text{m})$ **OR** $x=93.4\ (93.3598\ldots)\ (\text{m})$ ***A1***

(width $=$) $124\ (124.479\ldots)\ (\text{m})$ ***A1***

(width $=$) $12.9\ (12.8534\ldots)\ (\text{m})$ ***A1***

Note: To award the final ***A1*** both values of x **and** both values of the width must be seen. Accept 12.8 for second value of width from candidate dividing 1200 by 3 sf value of 93.4.

[4 marks]

(c) $\left(\frac{\mathrm{d}A}{\mathrm{d}x}=\right)-4+\frac{3600}{x^2}$ **OR** $-4+3600x^{-2}$ ***A1A1A1***

Note: Award ***A1*** for -4, ***A1*** for $+3600$, and ***A1*** for x^{-2} or x^2 in denominator.

[3 marks]

(d) setting *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ equal to 0 **OR** sketch of *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ with x-intercept highlighted ***M1***

$(x=)\ 30\ (\text{m})$ ***A1***

Note: To award ***A1FT*** the candidate's value of x must be within the domain given in the problem $(3<x<300)$.

[2 marks]

(e) **EITHER**

evidence of using GDC to find maximum of graph of $A=1212-4x-\frac{3600}{x}$ ***(M1)***

OR

substitution of *their* x into A ***(M1)***

OR

dividing 1200 by *their* x to find width of park **and** subtracting 3 from *their* x and 4 from the width to find park dimensions ***(M1)***

Note: For the last two methods, only follow through if $3<$ their $x<300$.

THEN

$(A=)\ 972\ (\text{m}^2)$ ***A1***

[2 marks]

Total [16 marks]

Q19.

(a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ ***(M1)***

$(4, 1)$ ***A1***

Note: Award ***A0*** if parentheses are omitted in the final answer.

[2 marks]

(b) attempt to substitute values into gradient formula ***(M1)***

$\left(\frac{0-2}{6-2} = \right) -\frac{1}{2}$ ***(A1)***

therefore the gradient of perpendicular bisector is 2 ***(M1)***

so $y-1=2(x-4)$ $(y=2x-7)$ ***A1***

[4 marks]

(c) identifying the correct equations to use: ***(M1)***

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or of finding intersection point graphically ***(M1)***

$(3, -1)$ ***A1***

Note: Accept an answer expressed as "$x=3$, $y=-1$".

[3 marks]

(d) attempt to use distance formula ***(M1)***

$\mathrm{YZ} = \sqrt{(7-(-1))^2 + (7-3)^2}$

$= \sqrt{80}$ $\left(4\sqrt{5}\right)$ ***A1***

[2 marks]

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule ***(M1)***

$\cos \mathrm{X\hat{Y}Z} = \frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}$ $(=0.316227\ldots)$ ***(A1)***

Note: Award ***A1*** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} = \right) 71.6°$ $(71.5650\ldots°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ ***(M1)(A1)***

Note: Award ***A1*** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule.
Exact values do not need to be used in the substitution.

$\left(\hat{XYZ} =\right)$ $71.6°$ $(71.5650°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\ldots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ ***(M1)***

$= 24\ \text{km}^2$ ***A1***

[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc. ***R1***

[1 mark]

Total [18 marks]

Q20.

(a) (i) Let X be the random variable "distance from O".

$X \sim \mathrm{N}(10, 3^2)$

$\mathrm{P}(X < 13) = 0.841 \ (0.841344...)$ ***(M1)A1***

(ii) $\left(\mathrm{P}(X > 15) =\right) \ 0.0478 \ (0.0477903)$ ***A1***

[3 marks]

(b) $\mathrm{P}(X > 15) \times \mathrm{P}(X > 15)$ ***(M1)***

$= 0.00228 \ (0.00228391...)$ ***A1***

[2 marks]

(c) $1 - (0.8143)^3$ ***(M1)***

$0.460 \ (0.460050...)$ ***A1***

[2 marks]

(d) (i) **METHOD 1**

let Y be the random variable "number of points scored"

evidence of use of binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10, 0.539949...)$ ***(A1)***

$\left(\mathrm{P}(Y \geq 5) =\right) \ 0.717 \ (0.716650...)$. ***A1***

METHOD 2

let Q be the random variable "number of times a point is not scored"

evidence of use of binomial distribution ***(M1)***

$Q \sim \mathrm{B}(10, 0.460050...)$ ***(A1)***

$\left(\mathrm{P}(Q \leq 5) =\right) \ 0.717 \ (0.716650...)$ ***A1***

(ii) $\mathrm{P}(5 \leq Y < 8)$ ***(M1)***

$0.628 \ (0.627788...)$ ***A1***

Note: Award ***M1*** for a correct probability statement or indication of correct lower and upper bounds, 5 and 7.

(iii) $\dfrac{\mathrm{P}(5 \leq Y < 8)}{\mathrm{P}(Y \geq 5)} \left(= \dfrac{0.627788...}{0.716650...}\right)$ ***(M1)***

$0.876 \ (0.876003...)$ ***A1***

[7 marks]

Total: [14 marks]

Q21.

(a) evidence of splitting diagram into equilateral triangles ***M1***

area $= 6\left(\frac{1}{2}x^2 \sin 60^\circ\right)$ ***A1***

$= \frac{3\sqrt{3}x^2}{2}$ ***AG***

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(b) total surface area of prism $1200 = 2\left(3x^2\frac{\sqrt{3}}{2}\right) + 6xh$ ***M1A1***

Note: Award ***M1*** for expressing total surface areas as a sum of areas of rectangles and hexagons, and ***A1*** for a correctly substituted formula, equated to 1200.

$h = \frac{400 - \sqrt{3}x^2}{2x}$ ***A1***

volume of prism $= \frac{3\sqrt{3}}{2}x^2 \times h$ ***(M1)***

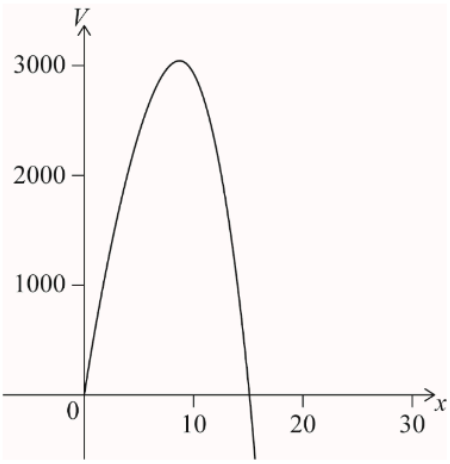
$= \frac{3\sqrt{3}}{2}x^2\left(\frac{400 - \sqrt{3}x^2}{2x}\right)$ ***A1***

$= 300\sqrt{3}x - \frac{9}{4}x^3$ ***AG***

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[5 marks]

(c)



A1A1

Note: Award ***A1*** for correct shape, ***A1*** for roots in correct place with some indication of scale (indicated by a labelled point).

[2 marks]

(d) $\frac{dV}{dx} = 300\sqrt{3} - \frac{27}{4}x^2$ ***A1A1***

Note: Award ***A1*** for a correct term.

[2 marks]

(e) from the graph of V or $\frac{dV}{dx}$ **OR** solving $\frac{dV}{dx} = 0$ ***(M1)***

$x = 8.77$ $(8.77382...)$ ***A1***

[2 marks]

(f) from the graph of V **OR** substituting their value for x into V ***(M1)***

$V_{max} = 3040 \text{ cm}^3$ $(3039.34...)$ ***A1***

[2 marks]

(g) **EITHER**

wasted space / spheres do not pack densely (tesselate) ***A1***

OR

the model uses exterior values / assumes infinite thinness of materials and hence the modelled volume is not the true volume ***A1***

[1 mark]

Total [16 marks]

Q22.

Markscheme
$\frac{360}{12}$ ***M1*** $= 30$ ***AG*** **Note:** Award ***M0*** for use of given value, 30. ***[1 mark]***

(b)

Markscheme
substitution of $t = 5$ into model $(-2.5\cos(30 \times 5) + 4.5)$ ***(M1)*** $h(5) = 6.67\ (6.66506\ldots)$ ***A1*** **Note:** Award ***M1A0*** for $2.75187\ldots$ (radian answer). ***[2 marks]***

(c) (c.i)

Markscheme
$2.5\ (m)$ ***A1*** **Note:** Award ***A0*** for $-2.5\ (m)$ ***[1 mark]***

(c.ii)

Markscheme
$h(t) = 4.5$ ***A1A1*** **Note:** Award ***A1*** for an equation of a horizontal line $h =$ constant (accept $y =$ constant), ***A1*** for that constant being 4.5. Second ***A1*** is dependent on first ***A1***. Do not award ***A0A1***. ***[2 marks]***

(d)

Markscheme
attempt at finding point of intersection between $h(t) = 2.65$ and the curve. ***(M1)*** $13.4\ (13.4089\ldots)$ ***(A1)*** $13{:}25\ (1{:}25\ p.m.)$ ***A1*** **Note:** Award ***M1A1A1FT*** for consistent use of radians $12.1720\ldots$ $12{:}10,\ 0{:}10\ p.m.$

Award at most ***M1A0A0*** if the student works in the first cycle ($t = 1.40895\ldots$)
[3 marks]

(e)

Markscheme
$22.5910\ldots - 13.4089\ldots - 0.5$ ***(A1)(M1)(M1)*** **Note:** Award ***A1*** for $22.5910\ldots$ seen, ***M1*** for subtracting their two times for harbour, ***M1*** for subtracting 0.5 from their answer at any stage of the work. 8.68 (hours) $(8.68209\ldots)$ (hours) ***A1*** ***[4 marks]*** **Note:** Award at most ***A0M0M1A0*** for consistent use of radians. There is no 15 minute window for the ship to reach nor return from the fishing site.

Q23. (a.i)

Markscheme
maximum $h = 130$ metres ***A1*** ***[1 mark]***

Markscheme
minimum $h = 50$ metres ***A1*** ***[1 mark]***

(b.i)

Markscheme
$(60 \div 12 =)$ 5 seconds ***A1*** ***[1 mark]***

(b.ii)

Markscheme
$360 \div 5$ ***(M1)***
Note: Award ***(M1)*** for 360 divided by their time for one revolution.
$= 72°$ ***A1***
[2 marks]

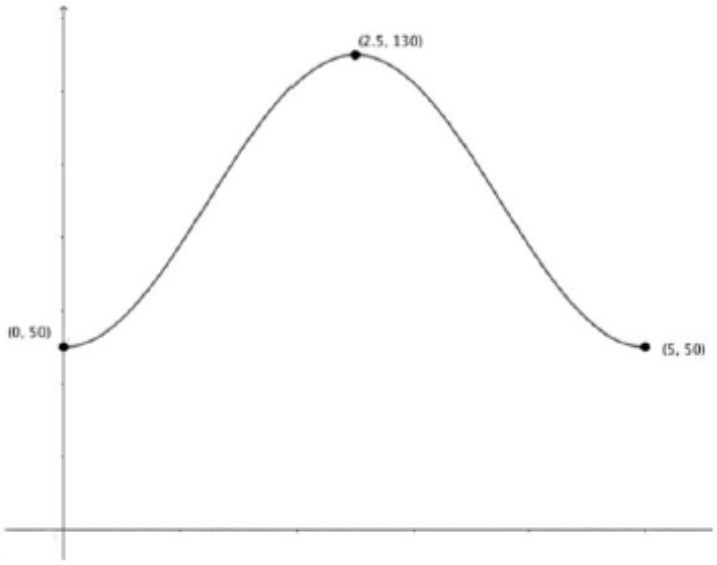
(c.i)

Markscheme
(amplitude =) 40 ***A1***
[1 mark]

(c.ii)

Markscheme
(period $= \frac{360}{72} =$) 5 ***A1***
[1 mark]

(d)

Markscheme


Maximum point labelled with correct coordinates. ***A1***
At least one minimum point labelled. Coordinates seen for any minimum points must be correct. ***A1***
Correct shape with an attempt at symmetry and “concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain. ***A1***

[3 marks]

(e.i)

Markscheme

$h = 90 - 40\cos(144°)$ ***(M1)***
$(h =)\ 122\,\text{m}\ \ (122.3606\ldots)$ ***A1***

[2 marks]

(e.ii)

[3]

Markscheme

evidence of $h = 100$ on graph **OR** $100 = 90 - 40\cos(72t)$ ***(M1)***
t coordinates $3.55\ (3.54892\ldots)$ **OR** $1.45\ (1.45107\ldots)$ or equivalent ***(A1)***

Note: Award ***A1*** for either t-coordinate seen.

$= 2.10$ seconds $(2.09784\ldots)$ ***A1***

[3 marks]

(f.i)

Markscheme

$5 - 2.09784\ldots$ ***(M1)***
$\frac{(2.902153\ldots)}{5}$ ***(M1)***
$0.580\ \ (0.580430\ldots)$ ***A1***

[3 marks]

(f.ii)

Markscheme

METHOD 1

changing the frequency/dilation of the graph will not change the proportion of time that point C is visible. ***A1***

$0.580\ \ (0.580430\ldots)$ ***A1***

METHOD 2

correct calculation of relevant found values

$\frac{(2.902153\ldots)/2}{5/2}$ ***A1***

$0.580\ \ (0.580430\ldots)$ ***A1***

Note: Award ***A0A1*** for an unsupported correct probability.

[2 marks]