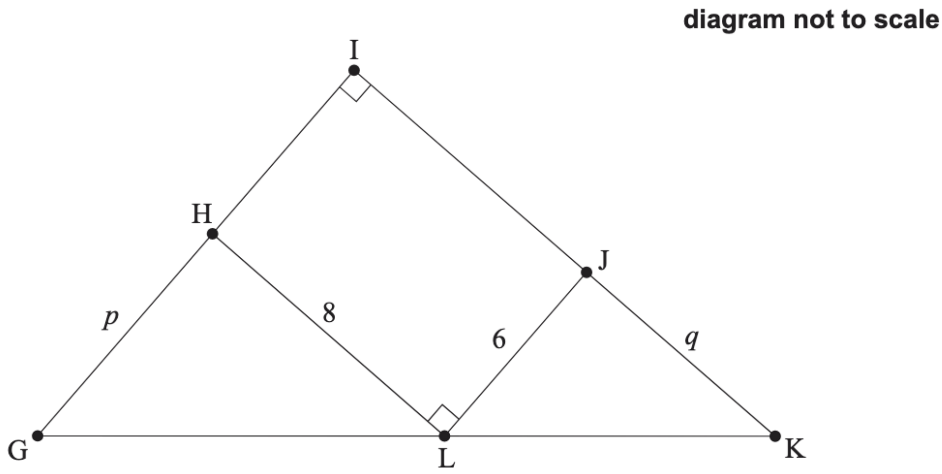
Q1. Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.



The area of the top of the gift box is A cm^2.

(a) (i) Find A in terms of p and q .

(ii) Show that $A = \frac{192}{q} + 3q + 48$.

(b) Find $\frac{dA}{dq}$.

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q .

(ii) Hence, or otherwise, find this value of q.

Q2. A company produces and sells electric cars. The company's profit, P , in thousands of dollars, changes based on the number of cars, x , they produce per month.

The rate of change of their profit from producing x electric cars is modelled by

$$\frac{dP}{dx} = -1.6x + 48, \qquad x \geq 0$$

The company makes a profit of 260 (thousand dollars) when they produce 15 electric cars.

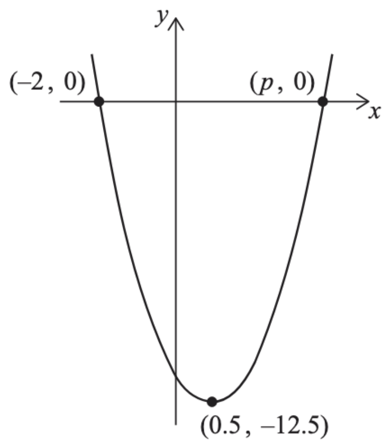
(a) Find an expression for P in terms of x .

The company regularly increases the number of cars it produces.

(b) Describe how their profit changes if they increase production to over 30 cars per month and up to 50 cars per month. Justify your answer.

Q3. Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates (0.5 , −12.5) . The graph intersects the x-axis at two points, (−2 , 0) and (p , 0) .

diagram not to scale



(a)Find the value of p .

(b) Find the value of

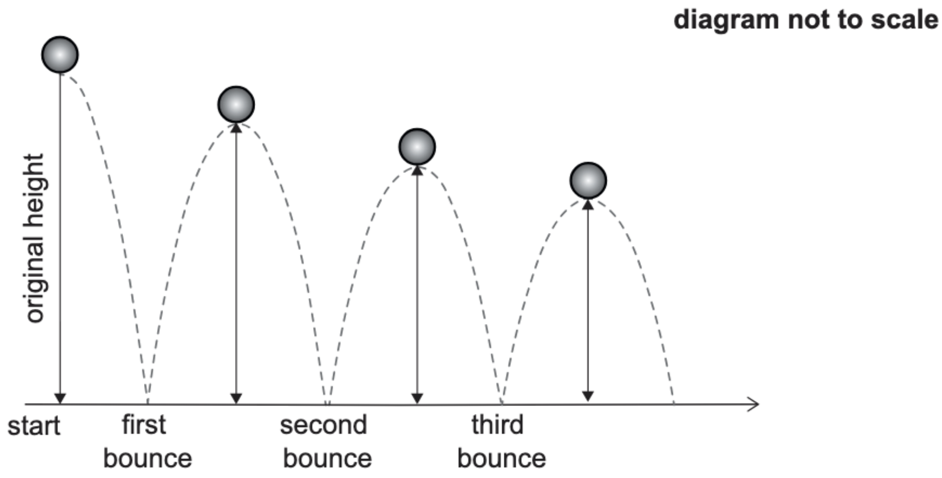
(i) a .

(ii) b .

(iii) c.

(c) Write down the equation of the axis of symmetry of the graph.

Q4. A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85 % of the previous maximum height.



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is 68 cm, to the nearest cm.

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than 10 cm.

(c) Find the total vertical distance travelled by the ball from the point at which it is dropped until the fourth bounce.

Q5. A company's profit per year was found to be changing at a rate of

$$\frac{dP}{dt} = 3t^2 - 8t$$

where P is the company's profit in thousands of dollars and t is the time since the company was founded, measured in years.

(a) Determine whether the profit is increasing or decreasing when $t = 2$.

One year after the company was founded, the profit was 4 thousand dollars.

(b) Find an expression for $P(t)$, when $t \geq 0$.

Q6. The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N , which have a magnitude of at least M . For a particular region the equation is

$\log_{10} N = a - M$, for some $a \in \mathbb{R}$.

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a.

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b.

(c) Given $0 < M < 8$, find the range for N.

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(d) Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year.

Q7. The aircraft for a particular flight has 72 seats. The airline's records show that historically for this flight only 90 % of the people who purchase a ticket arrive to board the flight.

They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

(a) The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight.

(b) (i) Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold.

(ii) Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72.

Each passenger pays $150 for a ticket. If too many passengers arrive, then the airline will give $300 in compensation to each passenger that cannot board.

(c) Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72.

Q8. Juliana plans to invest money for 10 years in an account paying 3.5 % interest, compounded annually. She expects the annual inflation rate to be 2 % per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $4000, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $1000 at the start of the 10-year period and then invest \$ x into the account at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar.

(b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar.

Q9. Consider the function $f(x) = x^2 - \frac{3}{x}, x \neq 0$

(a) Find $f'(x)$.

Line L is a tangent to $f(x)$ at the point (1 , −2).

(b) Use your answer to part (a) to find the gradient of L .

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer.

Q10. A boat travels 8 km on a bearing of 315° and then a further 6 km on a bearing of 045° .

Find the bearing on which the boat should travel to return directly to the starting point.

Q11. On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of 67.3 kmh^{-1} .

A speed of 75.7 kmh^{-1} is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars.

Speeding tickets are issued to all drivers travelling at a speed greater than 72 km h^{-1} .

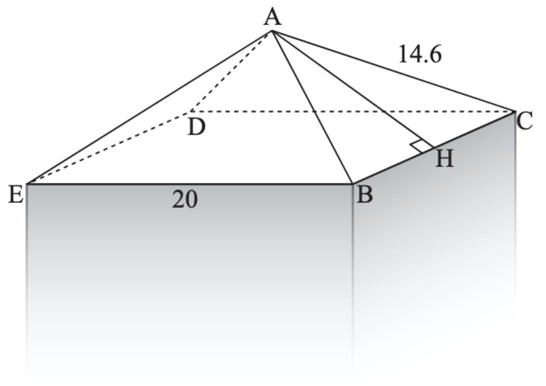
(b) Find the probability that a randomly selected driver who passes the speed camera receives a speeding ticket.

It is found that 82 % of cars on this road travel at speeds between p kmh^{-1} and q kmh^{-1} , where $p < q$. This interval includes cars travelling at a speed of 74 kmh^{-1}.

(c) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean.

Q12. Vertical posts are to be placed around the outer edge of a children's park. Each post is formed from a cuboid with a right square-based pyramid on top.

The cuboid part of the post is machine-made such that its width, and hence the width of the pyramid, is exactly 20 cm . The length from the apex of the pyramid, A , to any corner of the base of the pyramid is 14.6 cm , but this is only accurate to the nearest tenth of a centimetre. The post is shown in the diagram.



(a) Write down the upper bound and lower bound for the possible lengths of edge AC .

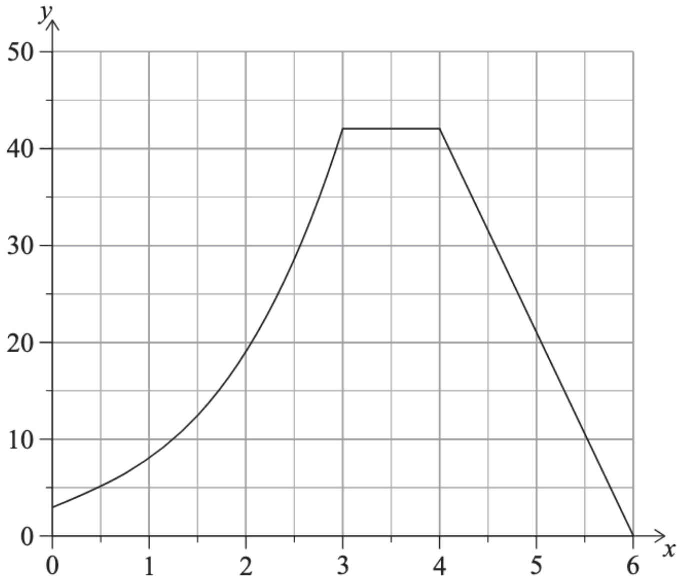
Point H is the midpoint of BC .

(b) Determine the upper bound and lower bound for AH , the slant height of the pyramid.

For the post to be safe for children, the angle between the slant height and the base of the pyramid must be less than 22° .

(c) Show that this post is safe for children. Justify your answer.

Q13. An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.



The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \leq x \leq 3$.

x	0	0.5	1	1.5	2	2.5	3
$y = f(x)$	3	5.13	8	12.4	19	28.6	42

a) Calculate an estimate for the area in the interval $0 \leq x \leq 3$ by using the trapezoidal rule with three equal intervals.

It is known that $f'(x) = 3x^2 + 4$ in the domain $0 < x < 3$.

(b) Find an expression for $f(x)$, in the domain $0 < x < 3$.

(c) Hence find the actual area of the entire cross-section.

Q14. In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$P(X=x)$	0.15	0.2	k	0.16	$2k$	0.25

a) Find the value of k.

The player has a chance to win money based on how many times they hit the target. The gain for the player, in $, is shown in the following table, where a negative gain means that the player loses money.

x	0	1	2	3	4	5
Player's gain ($)	–4	–3	–1	0	1	4

(b) Determine whether this game is fair. Justify your answer.

Q15. A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and $a,\ b,\ c \in \mathbb{R}^+$.

When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c.

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b.i) Find the value of a.

(b.ii) Find the value of b.

(c) Write down the minimum height of point P.

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b.

Q16. Nicole works at a local school 5 days each week. She drives an old car to work that has a 72 % probability of starting on any given morning. The probability of the car starting on a given morning is independent of it starting on any other morning.

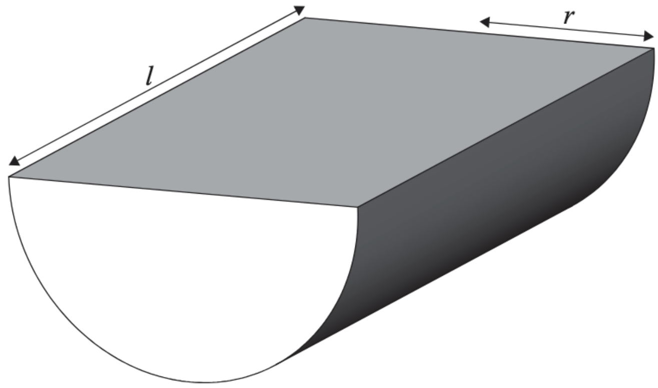
(a) Find the probability that Nicole's car starts on exactly three mornings in a particular 5 day workweek.

Nicole walks to work on mornings when her car does not start and it is not raining.
Nicole takes the bus to work on mornings when her car does not start and it is raining.
Where Nicole lives, there is a 42 % probability of rain on any given morning, independent of any other morning. The probability of Nicole's car starting is independent of the weather.

(b) Find the probability that Nicole will not have to take the bus in a particular workweek.

Q17. A large closed container, in the shape of a half cylinder with a rectangular lid, is to be constructed with a volume of 0.8 m^3. The container has a length of l metres and a radius of r metres.

diagram not to scale



(a) Find an exact expression for l in terms of r and π.

The container will be constructed using two different materials. The material for both the curved surface and the rectangular lid of the container costs \$ 4.40 per square metre.

The material for the semicircular ends of the container costs \$ p per square metre.

The cost, C , of the materials to construct the container can be written in terms of r and p (where $p > 0$ and $r > 0$).

(b) Show that $C = 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$.

(c) Find $\frac{dC}{dr}$.

The cost of materials to construct the container is minimized when the radius of the container, r , is 0.7 m .

(d) Find the value of p .

In total, 350 containers will be constructed at this minimum cost.

(e) Calculate the cost of materials, to the nearest dollar, to construct all 350 containers.

The materials for constructing the containers can be purchased at a discount according to the information in the table.

Cost of materials (\$ C) before discount	**Discount applied to entire order**
$1000 \le C < 2500$	1%
$2500 \le C < 5000$	4%
$5000 \le C < 10\,000$	8%
$C \ge 10\,000$	10%

(f) Determine the cost of materials for 350 containers after the discount is applied.

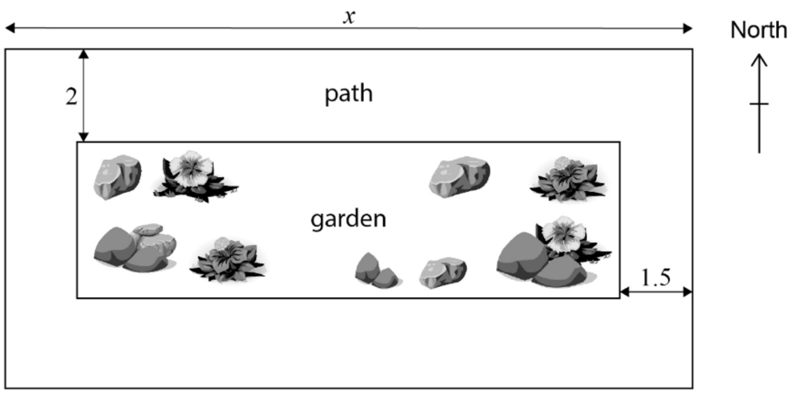
Q18. A particular park consists of a rectangular garden, of area A m^2, and a concrete path surrounding it. The park has a total area of 1200 m^2.

The width of the path at the north and south side of the park is 2 m.

The width of the path at the west and east side of the park is 1.5 m.

The length of the park (along the north and south sides) is x metres, $3 < x < 300$.

diagram not to scale



(a) (i) Write down the length of the garden in terms of x .

(ii) Find an expression for the width of the garden in terms of x .

(iii) Hence show that $A = 1212 - 4x - \frac{3600}{x}$

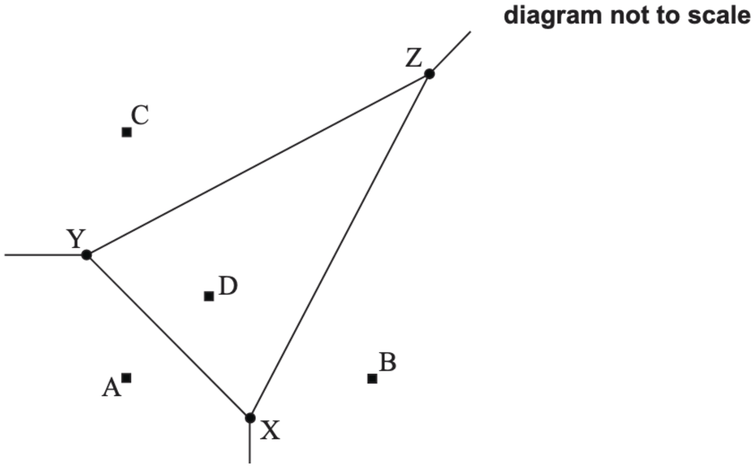
(b) Find the possible dimensions of the park if the area of the garden is 800 m^2.

(c) Find an expression for $\frac{dA}{dx}$.

(d) Use your answer from part (c) to find the value of x that will maximize the area of the garden.

(e) Find the maximum possible area of the garden.

Q19. The Voronoi diagram below shows four supermarkets represented by points with coordinates A(0 , 0) , B(6 , 0), C(0 , 6) and D(2 , 2). The vertices X, Y, Z are also shown. All distances are measured in kilometres.



(a) Find the midpoint of [BD].

(b) Find the equation of (XZ).

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X.

The coordinates of Y are (−1 , 3) and the coordinates of Z are (7 , 7).

(d) Determine the exact length of [YZ].

(e) Given that the exact length of [XY] is $\sqrt{32}$, find the size of $X\hat{Y}Z$ in degrees.

(f) Hence find the area of triangle XYZ.

A town planner believes that the larger the area of the Voronoi cell XYZ, the more people will shop at supermarket D.

(g) State one criticism of this interpretation.

Q20. Arianne plays a game of darts.

The distance that her darts land from the centre, O, of the board can be modelled by a normal distribution with mean 10 cm and standard deviation 3 cm.

(a) Find the probability that

(i) a dart lands less than 13 cm from O.

(ii) a dart lands more than 15 cm from O.

Each of Arianne's throws is independent of her previous throws.

(b) Find the probability that Arianne throws two consecutive darts that land more than 15 cm from O.

In a competition a player has three darts to throw on each turn. A point is scored if a player throws all three darts to land within a central area around O. When Arianne throws a dart the probability that it lands within this area is 0.8143.

(c) Find the probability that Arianne does not score a point on a turn of three darts.

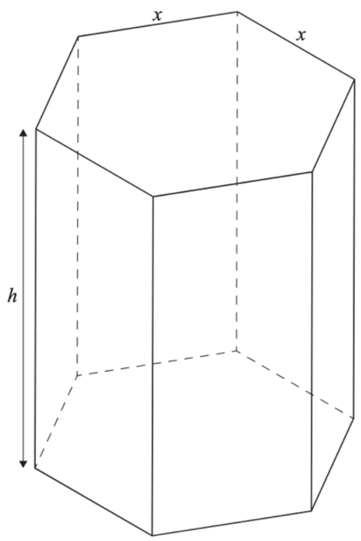
In the competition Arianne has ten turns, each with three darts.

(d) (i) Find the probability that Arianne scores at least 5 points in the competition.

(ii) Find the probability that Arianne scores at least 5 points and less than 8 points.

(iii) Given that Arianne scores at least 5 points, find the probability that Arianne scores less than 8 points.

Q21. A hollow chocolate box is manufactured in the form of a right prism with a regular hexagonal base. The height of the prism is h cm, and the top and base of the prism have sides of length x cm.



(a) Given that $\sin 60° = \frac{\sqrt{3}}{2}$, show that the area of the base of the box is equal to $\frac{3\sqrt{3}x^2}{2}$.

(b) Given that the total external surface area of the box is 1200 cm2, show that the volume of the box may be expressed as

(c) Sketch the graph of $V = 300\sqrt{3}x - \frac{9}{4}x^3$, for $0 \le x \le 16$.

(d) Find an expression for $\frac{dV}{dx}$.

(e) Find the value of x which maximizes the volume of the box.

(f) Hence, or otherwise, find the maximum possible volume of the box.

The box will contain spherical chocolates. The production manager assumes that they can calculate the exact number of chocolates in each box by dividing the volume of the box by the volume of a single chocolate and then rounding down to the nearest integer.

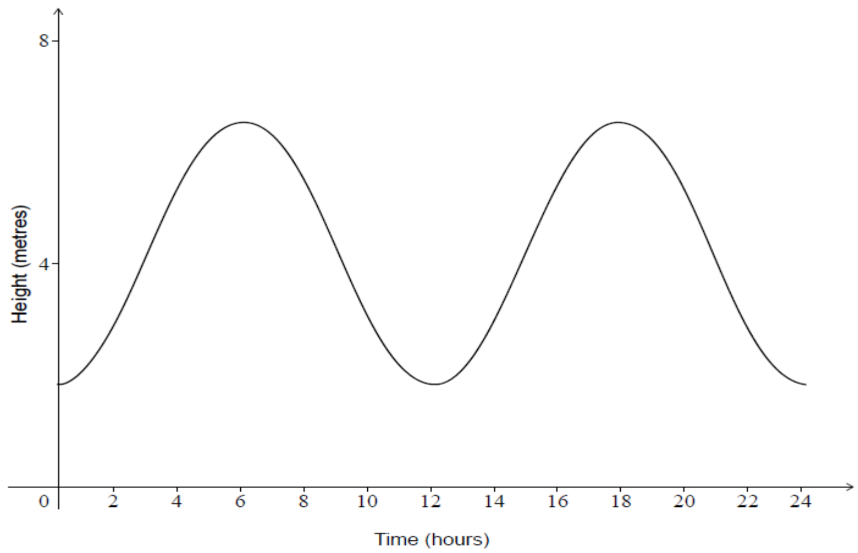
(g) Explain why the production manager is incorrect.

Q22. On a particular day the height of the tide, h, in metres, at Albion harbour can be modelled by the function

$$h(t) = -2.5\cos(bt^\circ) + 4.5, \text{ where } b \in \mathbb{R},\ 0 \leq t \leq 24$$

and t represents the number of hours after midnight.

The graph of h is shown in the following diagram.



(a) Show that the value of b is 30.

(b) Find the height of the tide when $t = 5$.

(c) Write down

(c.i) the amplitude of h.

(c.ii) the equation of the principal axis.

Boats can only leave or return to Albion harbour when $h(t) \geq 2.65$. Robin wants to leave the harbour to go fishing as soon as possible after the time is $12{:}00$.

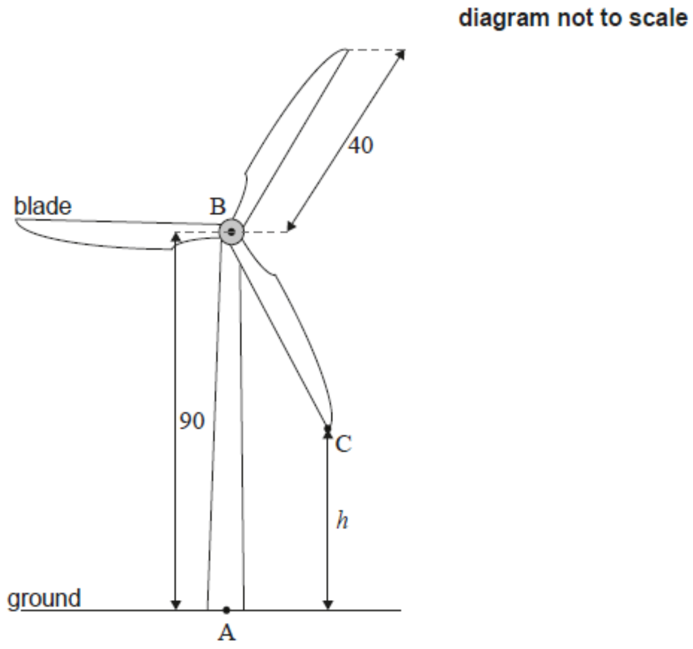
(d) Determine the earliest possible time that Robin could leave the harbour. Give your answer to the nearest minute.

The boat will take 15 minutes to travel from the harbour to the fishing site. Robin intends to return to the harbour on the same day.

(e) Determine the maximum length of time he could spend at the fishing site, in hours, and still be certain he will be able to enter the harbour on his return.

Q23. A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is 90 m. The blades of the turbine are centred at B and are each of length 40 m. This is shown in the following diagram.



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

Find the

(a.i) maximum value of h.

(a.ii) minimum value of h.

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

(b.i) Find the time, in seconds, it takes for the blade [BC] to make one complete rotation under these conditions.

(b.ii) Calculate the angle, in degrees, that the blade [BC] turns through in one second.

The height, h, of point C can be modelled by the following function. Time, t, is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40\cos(72t^\circ),\ t \geq 0$$

(c.i) Write down the amplitude of the function.

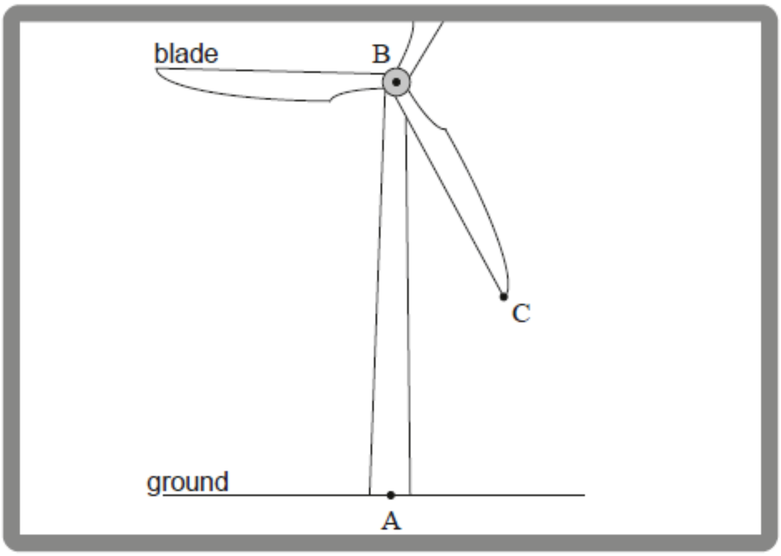
(c.ii) Find the period of the function.

(d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points.

(e.i) Find the height of C above the ground when $t = 2$.

(e.ii) Find the time, in seconds, that point C is above a height of 100 m, during each complete rotation.

Looking through his window, Tim has a partial view of the rotating wind turbine. The position of his window means that he cannot see any part of the wind turbine that is **more than** 100 m above the ground. This is illustrated in the following diagram.



(f.i) At any given instant, find the probability that point C is visible from Tim's window.

(f.ii) The wind speed increases. The blades rotate at twice the speed, but still at a constant rate.

At any given instant, find the probability that Tim can see point C from his window. Justify your answer.