MARKING SCHEME 2

1. 25M.1.SL.TZ1.6

(a.i)

Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1}\ cm^2$.

[2]

Markscheme
for a sequence of areas, uses two consecutive terms to find a common ratio **OR** for sequences of both widths and heights uses two consecutive terms for both sequences to find both common ratios **OR** recognises that both widths and heights are geometric sequences with common ratio $\frac{3}{2}$ ***M1*** areas form a geometric sequence with first term 20 and common ratio $\frac{45}{20}$ ***A1*** **OR** area of picture frame F_n is $4\left(\frac{3}{2}\right)^{n-1} \times 5\left(\frac{3}{2}\right)^{n-1}$ area of F_n is $20\left(\frac{9}{4}\right)^{n-1}$ ***AG***   ***[2 marks]***

(a.ii)

Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^{a}-1\right)\ cm^2$, where $p \in \mathbb{Q}^+, a \in \mathbb{Z}^+$.

[3]

Markscheme
attempt to find the sum of the areas using $S_n = \frac{u_1(r^n-1)}{r-1}$ ***(M1)*** sum of areas $\frac{20\left(\left(\frac{9}{4}\right)^{10}-1\right)}{\frac{9}{4}-1}\left(=16\left(\left(\frac{9}{4}\right)^{10}-1\right)\right)$ ***(A1)*** mean area $= \frac{1}{10}\left(\frac{20\left(\left(\frac{9}{4}\right)^{10}-1\right)}{\frac{9}{4}-1}\right)\left(=\frac{1}{10}\left(16\left(\left(\frac{9}{4}\right)^{10}-1\right)\right)\right)=\frac{16}{10}\left(\left(\frac{9}{4}\right)^{10}-1\right)\left(=\frac{8}{5}\left(\left(\frac{9}{4}\right)^{10}-1\right)\right)$ ***A1*** $p=\frac{8}{5}, a=10$ ***[3 marks]***

(b)

Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^4\ cm^2$, where $q \in \mathbb{Q}^+$.

[3]

Markscheme
recognition that median is between 5th and 6th picture frame ***(M1)*** median area $= \frac{20\left(\frac{9}{4}\right)^4+20\left(\frac{9}{4}\right)^5}{2}$ ***(A1)*** $= \frac{20\left(\frac{9}{4}\right)^4\left(1+\frac{9}{4}\right)}{2} = \frac{65}{2}\left(\frac{9}{4}\right)^4$ ***A1*** $q = \frac{65}{2}$ ***[3 marks]***

2. 25M.1.SL.TZ1.7

(a)

Determine the object's

[[N/A]]

(a.i)

maximum velocity;

[2]

Markscheme
attempt to find $v(1)$ ***(M1)*** $v(1) = 30 + 20 - 10$ max velocity 40 (ms^{-1}) ***A1*** ***[2 marks]***

(a.ii)

maximum speed.

[2]

Markscheme
attempt to find $v(5)$ ***(M1)*** $v(5) = 30 + 20(5) - 10(5)^2 = -120$ max speed 120 (ms^{-1}) ***A1*** ***[2 marks]***

(b.i)

Write down the value of T.

[1]

Markscheme
$T = 3$ ***A1*** ***[1 mark]***

(b.ii)

Find the distance travelled by the object in the first T seconds.

[4]

Markscheme
attempt to set up an integral of $v(t)$ to find area under velocity graph ***(M1)*** $\int_0^3 (30 + 20t - 10t^2)dt$ $\left[30t + 10t^2 - \frac{10t^3}{3}\right]_0^3$ ***A1*** attempt to substitute their limits into their integrated function and subtract ***(M1)*** $30(3) + 10(3)^2 - \frac{10(3)^3}{3} (= 90 + 90 - 90)$ distance 90 (metres) ***A1*** ***[4 marks]***

(c)

Determine whether the object returns to its initial position during the time period $0 \leq t \leq 5$, justifying your answer.

[4]

Markscheme

METHOD 1 attempt to find total displacement by setting up an integral of $v(t)$ between $t = 0$ and $t = 5$ ***(M1)*** $\int_0^5 (30 + 20t - 10t^2)\,dt = \left[30t + 10t^2 - \frac{10t^3}{3}\right]_0^5$ ***(A1)*** $= 30(5) + 10(5)^2 - \frac{10(5)^3}{3} = 400 - \frac{1250}{3}\left(= -\frac{50}{3}\right)$ ***A1*** total displacement is negative **OR** total displacement is not zero, (so the object does return to its initial position) ***R1*** **METHOD 2** attempt to find displacement after change in direction by setting up an integral of $v(t)$ between $t = 3$ and $t = 5$ ***(M1)*** $\int_3^5 (30 + 20t - 10t^2)dt = \left[30t + 10t^2 - \frac{10t^3}{3}\right]_3^5$ ***(A1)*** $= \left(30(5) + 10(5)^2 - \frac{10(5)^3}{3}\right) - \left(30(3) + 10(3)^2 - \frac{10(3)^3}{3}\right) = \left(400 - \frac{1250}{3}\right) - 90 = -\frac{50}{3} - 90\left(= -\frac{320}{3}\right)$ ***A1*** $\frac{320}{3} > 90$ compares to distance before change of direction, (so the object does return to its initial position) ***R1***
[4 marks]

3. 24N.1.SL.TZ1.6

[6]

Markscheme

attempt to use $u_n = u_1 + (n-1)d$ or $S_n = \frac{n}{2}[2u_1 + (n-1)d]$ or $S_n = \frac{n}{2}[u_1 + u_n]$ to set up at least one equation in u_1 and d ***(M1)*** $14 = u_1 + 9d$ and $200 = \frac{25}{2}[2u_1 + 24d]$ ***A1*** attempt to solve their two linear equations in u_1 and d simultaneously (must eliminate one variable) ***(M1)*** $d = -2$ $(\Rightarrow u_1 = 32)$ ***(A1)*** attempt to solve $u_k = 0$ with their d (or with their d and u_1) ***(M1)*** $\Rightarrow k = 17$ ***A1***
[6 marks]

4. 24N.1.SL.TZ1.7

(a)

Write down the equation for f in the form $f(x) = mx + c$.

[2]

Markscheme

$f(x) = -2x + 12$ (accept $y = -2x + 12$, accept $m = -2$ and $c = 12$) ***A1A1*** **Note:** Award ***A1*** for correct gradient, ***A1*** for correct y-intercept

[2 marks]

(b)

Find the value of b.

[3]

Markscheme

METHOD 1 Axis of Symmetry axis of symmetry is $\frac{-2+6}{2}$ $(= 2)$ ***(A1)*** equating $\frac{-b}{2a}$ to their axis of symmetry ***(M1)*** $\frac{-b}{-2} = 2$ $b = 4$ ***A1*** **METHOD 2** substitution attempt to substitute $(-2, 0)$ or $(6, 0)$ into g ***(M1)*** $-(-2)^2 - 2b + 12 = 0$ or $-(6)^2 + 6b + 12 = 0$ ***(A1)*** $b = 4$ ***A1*** **METHOD 3** factored form attempt to write g in factored form ***(M1)*** $g(x) = (x + 2)(6 - x)$ $-x^2 + 4x + 12$ ***(A1)*** $b = 4$ ***A1*** **METHOD 4** quadratic formula attempt to substitute into quadratic formula and set equal to -2 or 6 ***(M1)*** $6 = \frac{-b \pm \sqrt{b^2 - 4(-1)(12)}}{2(-1)}$ OR $-2 = \frac{-b \pm \sqrt{b^2 - 4(-1)(12)}}{2(-1)}$ $b^2 - 24b + 144 = b^2 + 48$ OR $b^2 + 8b + 16 = b^2 + 48$ (or equivalent, must not contain radical) ***(A1)*** $b = 4$ ***A1***

[3 marks]

(c)

Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^6 (-x^2 + 6x)\, dx$.

[2]

Markscheme

recognizing to subtract $g - f$ (in correct order) ***(M1)*** $\int_0^6 (-x^2 + 4x + 12) - (-2x + 12) dx$ ***A1*** $\int_0^6 (-x^2 + 6x)\, dx$ ***AG***

[2 marks]

(d)

Hence, find the area of the region enclosed by the graph of f and the graph of g.

[4]

Markscheme

attempt to integrate ***(M1)*** $-\frac{1}{3}x^3 + 3x^2\ (+C)$ ***A1*** attempt to substitute limits into their integrated function and find difference ***(M1)*** $\left(-\frac{1}{3}\cdot 6^3 + 3\cdot 6^2\right) - \left(-\frac{1}{3}\cdot 0^3 + 3\cdot 0^2\right)\ 36$ ***A1***
[4 marks]

(e)

Find the coordinates of P.

[5]

Markscheme

$g'(x) = -2x + 4$ ***A1*** attempt to equate their derivative of g to their gradient of f ***(M1)*** $-2x + 4 = -2$ ***A1*** $x = 3$ ***A1*** $y = 15$ ***A1***
[5 marks]

5. 25M.1.SL.TZ2.6

(a)

Show that the distance, l, between the origin and any point on the graph of f is given by $l = \sqrt{x^2 \ln x + 4}$.

[1]

Markscheme

correct substitution into distance formula ***A1*** $l^2 = x^2 \ln x + 4 - x^2 + x^2$ **OR** $\sqrt{(x-0)^2 + \left(\sqrt{x^2 \ln x + 4 - x^2} - 0\right)^2}$ **OR** $\sqrt{x^2 + x^2 \ln x + 4 - x^2}$ $l = \sqrt{x^2 \ln x + 4}$ ***AG***
[1 mark]

(b)

Hence, find the x-coordinate of the point on the graph of f which is closest to the origin.

[6]

Markscheme

recognising $\frac{dl}{dx} = 0$ (seen anywhere) ***(M1)***

EITHER attempt to use chain rule with l ***(M1)*** $\frac{1}{2}(x^2 \ln x + 4)^{-\frac{1}{2}} \times \frac{d}{dx}(x^2 \ln x + 4)$ attempt to use product rule with $\frac{d}{dx}(x^2 \ln x + 4)$ ***(M1)*** $\frac{1}{2}(x^2 \ln x + 4)^{-\frac{1}{2}} \times \left[x^2 \times \frac{1}{x} + \ln x \times 2x\right]$ ***A1***

OR recognising to minimise $x^2 \ln x + 4$ ***(M1)*** attempt to use product rule ***(M1)*** $x^2 \times \frac{1}{x} + \ln x \times 2x$ ***A1***

THEN $x + 2x \ln x = 0$ (or equivalent) ***A1*** $\Rightarrow \ln x = -\frac{1}{2}$ $x = e^{-\frac{1}{2}}$ $\left(= \frac{1}{\sqrt{e}}\right)$ ***A1***

Note: Award ***A0*** for including $x = 0$ in the final answer.
[6 marks]

6. 25M.1.SL.TZ2.7

(a)

Show that $k = \frac{1}{3}$.

[5]

Markscheme

recognise sum of probabilities is 1 (seen anywhere) ***(M1)*** $k + 3k^2 + 2k^2 + k^2 = 1$ **OR** $\sum P(X = x) = 1$

$6k^2 + k - 1 = 0$ **OR** $6k^2 + k = 1$ ***A1***

valid attempt to solve their quadratic $= 0$ ***(M1)***

$(3k - 1)(2k + 1)$ **OR** $k = \frac{-1 \pm \sqrt{1-4(6)(-1)}}{2(6)}$

$k = \frac{1}{3}$ and $k = -\frac{1}{2}$ ***A1***

$0 < k < 1$ (so, disregard $k = -\frac{1}{2}$) ***R1*** **Note:** accept $k > 0$ or $k \neq 0$ or k is a probability.
Do not award ***R1*** unless the previous mark has been awarded. $k = \frac{1}{3}$ ***AG*** **Note:** If a candidate uses $k = \frac{1}{3}$ to verify that the probabilities sum to 1, award ***M1A0M0A0R0***.
[5 marks]

(b)

Find $P(X < 3a)$.

[2]

Markscheme

$1 - P(X = 3a)$ **OR** $P(X = 0) + P(X = a) + P(X = 2a)$ ***(M1)***

$1 - k^2 \left(= 1 - \frac{1}{9}\right)$ **OR** $5k^2 + k \left(= \frac{5}{9} + \frac{1}{3}\right)$ **OR** $\frac{1}{3} + \frac{3}{9} + \frac{2}{9}$

$P(X < 3a) = \frac{8}{9}$ ***A1***
[2 marks]

(c)

Find $P(X \geq a \ \ X < 3a)$.

[3]

Markscheme
valid attempt to identify the correct required outcomes ***(M1)*** $\frac{P(X \geq a \cap X < 3a)}{P(X<3a)}$ **OR** $P(a \leq X < 3a)$ **OR** $P(X = a) + P(X = 2a)$ **OR** $X = a$ and $2a$ **OR** $P(X < 3a) - P(X < a)$ correct numerator $3k^2 + 2k^2 (= 5k^2)$ **OR** $\frac{3}{9} + \frac{2}{9}$ **OR** $\frac{8}{9} - \frac{1}{3}\left(= \frac{5}{9}\right)$ ***(A1)*** $\frac{\frac{5}{9}}{\frac{8}{9}} P(X \geq a \ X < 3a) = \frac{5}{8}$ ***A1*** ***[3 marks]***

(d)

Given that $E(X) = 20$, find the value of a.

[3]

Markscheme
attempt to use the expected value formula ***(M1)*** $0 \times \frac{1}{3} + a \times \frac{1}{3} + 2a \times \frac{2}{9} + 3a \times \frac{1}{9} \ (= 20) \ \frac{10a}{9} = 20$ **OR** $3a + 4a + 3a = 180$ (or equivalent) ***A1*** $a = 18$ ***A1*** ***[3 marks]***

7. 24M.1.SL.TZ1.6

(a)

Find an expression for S_n in the form $\frac{a^n - 1}{b}$, where $a, \ b \in \mathbb{Z}^+$.

[1]

Markscheme
$S_n = \frac{10^n - 1}{9}$ ***A1*** $(a = 10, b = 9)$ ***[1 mark]***

(b)

Hence, show that $S_1 + S_2 + S_3 + \cdots + S_n = \frac{10(10^n - 1) - 9n}{81}$.

[4]

Markscheme

METHOD 1 $S_1 + S_2 + S_3 + \cdots + S_n = \frac{10-1}{9} + \frac{10^2-1}{9} + \cdots + \frac{10^n-1}{9}$ ***(A1)*** $= \frac{10-1+10^2-1+10^3-1+\cdots+10^n-1}{9}$ OR $\frac{9(10-1+10^2-1+10^3-1+\cdots+10^n-1)}{81}$ attempt to use geometric series formula on powers of 10, and collect -1's together ***M1*** $10 + 10^2 + 10^3 + \cdots + 10^n = \frac{10(10^n-1)}{10-1}$ and $-1 - 1 - 1 \ldots = -n$ ***A1*** $= \frac{\frac{10(10^n-1)}{10-1} - n}{9}$ OR $\frac{9\left(\frac{10(10^n-1)}{10-1}\right) - 9n}{81}$ ***A1*** **Note:** Award ***A1*** for any correct intermediate expression. $= \frac{10(10^n-1)-9n}{81}$ ***AG*** **METHOD 2** attempt to create sum using sigma notation with S_n ***M1*** $\sum_{i=1}^{n} \frac{10^i-1}{9}\left(= \frac{1}{9}\left(\sum_{i=1}^{n} 10^i - \sum_{i=1}^{n} 1\right)\right) \sum_{i=1}^{n} 10^i = \frac{10(10^n-1)}{9}$ ***A1*** $\sum_{i=1}^{n} 1 = n$ ***A1*** $= \frac{1}{9}\left(\frac{10(10^n-1)}{9} - n\right)$ OR $\frac{1}{9}\left(\frac{10(10^n-1)-9n}{9}\right)$ ***A1*** $= \frac{10(10^n-1)-9n}{81}$ ***AG*** ***[4 marks]***

8. 24M.1.SL.TZ1.7

(a)

Find

[[N/A]]

(a.i)

$\frac{dy}{dx}$;

[2]

Markscheme

$\frac{dy}{dx} = 3x^2 - 2x - 1$ ***A1A1*** **Note:** Award ***A1*** for $3x^2 - 2x$ and ***A1*** for -1.
[2 marks]

(a.ii)

$\frac{d^2y}{dx^2}$.

[1]

Markscheme

$\frac{d^2y}{dx^2} = 6x - 2$ ***A1***
[1 mark]

(b)

Find the coordinates of A, using your answer to part (a)(ii) to justify your answer.

[6]

Markscheme

setting their (quadratic) $\frac{dy}{dx}$ to 0 and solve using valid method ***(M1)*** $(3x + 1)(x - 1) = 0$ OR $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} \left(= \frac{2 \pm \sqrt{4+12}}{6}\right)$ $x = -\frac{1}{3}$ (OR $x = 1$) ***A1*** substituting one of their x values into $\frac{d^2y}{dx^2}$ ***M1*** **EITHER** at $x = -\frac{1}{3}, \frac{d^2y}{dx^2} = 6\left(-\frac{1}{3}\right) - 2(= -4) < 0$ so local max ***R1*** **OR** at $x = 1, \frac{d^2y}{dx^2} = 6(1) - 2(= 4) > 0$ so local min hence local max at $x = -\frac{1}{3}$ ***R1*** **Note:** Award ***R1*** only if the previous ***M1*** has been awarded and there is reference to < 0 or > 0, as appropriate. **THEN** substituting their x-coordinate of A into y ***(M1)*** $y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 1$ $y = \frac{32}{27}$ ***A1*** so coordinates of A are $\left(-\frac{1}{3}, \frac{32}{27}\right)$ **Note:** This ***(M1)A1*** may be awarded independently of the previous ***M1R1***.
[6 marks]

(c)

Find the x-coordinate of B.

[2]

Markscheme
setting their $\frac{d^2y}{dx^2}$ to 0 ***(M1)*** $6x - 2 = 0$ $x = \frac{1}{3}$ ***A1*** ***[2 marks]***

(d)

Find the equation of L.

[3]

Markscheme
gradient of tangent $= -1$ ***(A1)*** negative reciprocal of their gradient ***(M1)*** gradient of normal $= 1$ equation is $y = x + 1$ (accept point/slope form $y - 1 = (x - 0)$) ***A1*** **Note:** Do not accept $L = x + 1$. ***[3 marks]***

9. 24M.1.SL.TZ2.6

(a)

Write down the equation of the horizontal asymptote.

[1]

Markscheme
$y = \frac{2}{3}$ (must be written as equation with $y =$) ***A1*** ***[1 mark]***

(b.i)

Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.

[1]

Markscheme

2 ***A1***
[1 mark]

(b.ii)

Determine the value of m such that $f(x) = g(x)$ has only one solution for x.

[4]

Markscheme

EITHER $\frac{2(x+3)}{3(x+2)} = mx + 1$ attempt to expand to obtain a quadratic equation ***(M1)*** $2x + 6 = 3mx^2 + 6mx + 3x + 6$ $3mx^2 + (6m + 1)x = 0$ OR $3mx^2 + 6mx + x = 0$ ***A1*** recognition that discriminant $\Delta = 0$ for one solution ***(M1)*** $(6m + 1)^2 = 0$ **OR** $\frac{2(x+3)}{3(x+2)} = mx + 1$ attempt to expand to obtain a quadratic equation ***(M1)*** $2x + 6 = 3mx^2 + 6mx + 3x + 6$ $3mx^2 + (6m + 1)x = 0$ OR $3mx^2 + 6mx + x = 0$ ***A1*** attempt to solve their quadratic for x and equating their solutions ***(M1)*** $x(3mx + 6m + 1) = 0$ $x = 0$ OR $x = -\frac{6m+1}{3m} (= 0)$ $-\frac{6m+1}{3m} = 0$ **OR** attempt to find $f'(x)$ using the quotient rule ***(M1)*** $f'(x) = \frac{2}{3}\left(\frac{(x+2)-(x+3)}{(x+2)^2}\right) = \left(\frac{-2}{3(x+2)^2}\right)$ OR $\frac{2(3x+6)-3(2x+6)}{(3x+6)^2}$ or equivalent ***A1*** recognition that m is the derivative of $f(x)$ at $x = 0$ ***(M1)*** **THEN** $\Rightarrow m = -\frac{1}{6}$ ***A1***
[4 marks]

(b.iii)

Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$.

[2]

Markscheme

Note: In this part, FT may be awarded only for values of m between -1 and 0. $-\frac{1}{6} < m < 0$ ***A2*** **Note:** Award ***A1*** for only $m > -\frac{1}{6}$. Award ***A1*** for only $m < 0$.
[2 marks]

10. 24M.1.SL.TZ2.7

(a)

Show that $b = \frac{\pi}{6}$.

[1]

Markscheme
$12b = 2\pi$ OR $(b =)\frac{2\pi}{12}$ OR $12 = \frac{2\pi}{b}$ ***A1*** $b = \frac{\pi}{6}$ ***AG***   ***[1 mark]***

(b)

Write down the value of

[[N/A]]

(b.i)

a;

[1]

Markscheme
$a = 5$ ***A1***   ***[1 mark]***

(b.ii)

c.

[1]

Markscheme
$c = 15$ ***A1***   ***[1 mark]***

(c)

Using this new model $g(x)$

[[N/A]]

(c.i)

find the midday water temperature on 1st October, five months after 1st May.

[3]

Markscheme
attempt to substitute $x = 5$ into $g(x)$ ***(M1)*** $g(5) = 3.5\sin\frac{5\pi}{6} + 11$ $\sin\frac{5\pi}{6} = \frac{1}{2}$ ***(A1)*** $g(5) = 3.5 \times \frac{1}{2} + 11$ $g(5) = 12.75\left(= \frac{51}{4}\right)$ ***A1***   ***[3 marks]***

(c.ii)

show that the midday water temperature is never warm enough for Alex to swim.

[3]

Markscheme
METHOD 1 (finding maximum temperature) considering the maximum value of $\sin\frac{\pi}{6}x(=1)$ OR $g'(x) = 0$ at maximum OR maximum = vertical shift + amplitude (may be seen on a graph) ***(M1)*** $g_{\max} = 3.5 + 11$ OR $\frac{\pi}{6} \cdot 3.5\cos\left(\frac{\pi}{6}x\right) = 0$ OR $x = 3$ $g_{\max} = 14.5$ ***A1*** $14.5 < 15$ (hence the midday water temperature is never warm enough for Alex to swim) ***R1*** **Note:** Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***). Worded conclusions are acceptable for the ***R1***, as long as the reasoning is clear that the water

does not reach $15°$, so not warm enough for Alex. **METHOD 2 (working with inequality)** $3.5\sin\left(\frac{\pi}{6}x\right)+11\geq 15$ ***(M1)*** $\sin\left(\frac{\pi}{6}x\right)\geq\frac{8}{7}$ ***A1*** sine values can never be greater than 1 (hence the midday water temperature is never warm enough for Alex to swim) ***R1*** **Note:** Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***). If candidate works with an equation throughout, the ***M1*** and ***A1*** may be awarded, if appropriate. A correct inequality is required for the ***R1*** to be awarded.
[3 marks]

(d)

Alex compares the two models and finds that $g(x)=0.7f(x)+q$. Determine the value of q.

[3]

Markscheme

EITHER attempt to find $0.7f(x)$ OR $0.7f(x)+q$ ***(M1)*** $0.7f(x)=3.5\sin\frac{\pi}{6}x+10.5$ OR $0.7f(x)+q=3.5\sin\frac{\pi}{6}x+10.5+q$ OR $10.5+q=11$ ***(A1)*** **OR** attempt to find $0.7f(x)$ for a particular value of x ***(M1)*** eg maximum $20\times 0.7=14$ ***(A1)*** **THEN** $q=0.5$ ***A1***
[3 marks]

11. 23N.1.SL.TZ1.6

[6]

Markscheme

attempt to apply binomial expansion ***(M1)*** $(1+kx)^n=1+{}^nC_1kx+{}^nC_2k^2x^2+\cdots$ OR ${}^nC_1k=\frac{9}{2}$ OR ${}^nC_2=15$ $nk=\frac{9}{2}$ ***(A1)*** $n\frac{(n-1)}{2}=15$ OR $\frac{n!}{(n-2)!2!}=15$ ***(A1)*** $(n^2-n-30=0)$ OR $n(n-1)=30$ valid attempt to solve ***(M1)*** $(n-6)(n-5)=0$ OR $6(6-1)=30$ OR finding correct value in Pascal's triangle $\Rightarrow n=6$ ***A1*** $\Rightarrow k=\frac{3}{4}$ ***A1***
Note: If candidate finds $n=6$ with no working shown, award ***M1A0A0M1A1A0***.

If candidate finds $n = 6$ and $k = \frac{3}{4}$with no working shown, award ***M1A0A0M1A1A1***.
[6 marks]

12. 23N.1.SL.TZ1.7

[[N/A]]

(a)

[N/A]

[[N/A]]

(a.i)

Find the value of p.

[1]

Markscheme	
$p = 9$ ***[1 marks]***	***A1***

(a.ii)

Write down the modal class.

[1]

Markscheme	
$600 < n \leq 800$ ***[1 marks]***	***A1*** **Note:** Award ***A0*** if candidate answers 700.

(b)

Use the cumulative frequency curve to estimate

[[N/A]]

(b.i)

the median number of tickets sold.

[1]

Markscheme
median $= 600$ ***A1***   ***[1 marks]***

(b.ii)

the number of performances where at least $80\,\%$ of the tickets were sold.

[3]

Markscheme
$80\,\%$ of $800 = 640$ ***(A1)*** 40 (performances less than $80\,\%$ of tickets sold) ***(A1)*** 20 (performances) ***A1***   ***[3 marks]***

(c)

[N/A]

[[N/A]]

(c.i)

State one disadvantage of the company surveying only the first $5\,\%$ of the audience as they leave the theatre.

[1]

Markscheme
any reasonable answer which suggests a biased sample (must include reason, do not accept reasons such as “sample size is too small”, or

answers that simply say “not representative of entire audience” without a valid reason) **A1** e.g. likely to come from the same part of the theatre OR be part of same group OR be from priority seating OR it is convenience sampling
[1 marks]

(c.ii)

Describe briefly how the company could collect feedback from $5\ \%$ of the audience using the systematic sampling method.

[2]

Markscheme
every 20^{th} person ***A1A1*** **Note:** Award ***A1*** for recognizing that sampling occurs at regular intervals e.g. “every”. Award ***A1*** for interval length is 20. ***[2 marks]***

(c.iii)

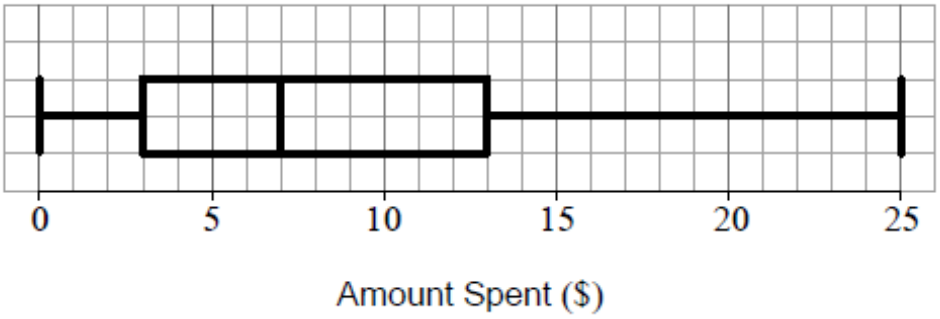
State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience.

[1]

Markscheme
quota (sampling method) ***A1*** ***[1 marks]***

(d)

The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.



[[N/A]]

(d.i)

Estimate the number of people who spent between $\$3$ and $\$25$.

[2]

Markscheme
$75\,\%$ (of 36000 spent between $\$3$ and $\$25$) ***(M1)*** $=$ 27000 ***A1*** ***[2 marks]***

(d.ii)

Half the audience spent less than $\$a$. Estimate the value of a.

[1]

Markscheme
$a = 7$ ***A1*** ***[1 marks]***

(e)

[N/A]

[[N/A]]

(e.i)

Calculate the mean number of tickets the company expects to sell this year for each performance.

[3]

Markscheme
METHOD 1 old mean is 600 (tickets) ***(A1)*** recognising new mean is old mean $+18$ ***(M1)*** $600 + 18 =$ 618 (tickets) ***A1*** **METHOD 2** new total number of tickets $=$ $36000 + 60 \times 18 (= 37080)$ ***(A1)*** $new\ mean = \frac{36000+60\times18}{60} \left(= \frac{37080}{60}\right)$ ***(M1)*** $= 618$ (tickets) ***A1*** ***[3 marks]***

(e.ii)

State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance.

[1]

Markscheme
no effect on the variance ***A1*** ***[1 marks]***

13. 23M.1.SL.TZ2.6

[7]

Markscheme
attempts to form $(g \circ f)(x)$ ***(M1)*** $[f(x)]^2 + f(x) + 3$ OR $(ax + b)^2 + ax + b + 3$ $a^2x^2 + 2abx + b^2 + ax + b + 3 (= 4x^2 - 14x + 15)$ ***(A1)*** equates their corresponding terms to form at least one equation ***(M1)*** $a^2x^2 = 4x^2$ OR $a^2 = 4$ OR $2abx + ax = -14x$ OR $2ab + a = -14$ OR $b^2 + b + 3 = 15$ $a = \pm 2$ (seen anywhere) ***A1*** attempt to use $2ab + a = -14$ to pair the correct values (seen anywhere) ***(M1)*** $f(x) = 2x - 4$ (accept $a = 2$ with $b = -4$), $f(x) = -2x + 3$ (accept $a = -2$ with $b = 3$) ***A1A1*** ***[7 marks]***

14. 23M.1.SL.TZ2.7

(a)

Write down the equation of the axis of symmetry.

[1]

Markscheme
$x = -2$ (must be an equation) ***A1 [1 mark]***

(b)

Write down the values of h and k.

[2]

Markscheme
$h = -2, k = -5$ ***A1A1 [2 marks]***

(c)

Find the y-coordinate of P.

[2]

Markscheme
substituting $x = 0$ into $f(x)$ ***(M1)*** $y = \frac{1}{4}(0+2)^2 - 5$ $y = -4$ (accept $P(0, -4)$) ***A1 [2 marks]***

(d)

Find the equation of the line L, in the form $y = ax + b$.

[3]

Markscheme

$f'(x) = \frac{1}{2}(x+2)$ $\left(= \frac{1}{2}x + 1\right)$ ***(A1)*** substituting $x = 0$ into their derivative ***(M1)*** $f'(0) = 1$ gradient of normal is -1 (may be seen in their equation) ***A1*** $y = -x - 4$ (accept $a = -1$, $b = -4$) ***A1*** **Note:** Award ***A0*** for $L = -x - 4$ (without the $y =$). ***[4 marks]***

(e)

Calculate the distance between P and Q.

[8]

Markscheme

equating their$f(x)$ to their L ***(M1)*** $\frac{1}{4}(x+2)^2 - 5 = -x - 4$ $\frac{1}{4}x^2 + 2x = 0$ (or equivalent) ***(A1)*** valid attempt to solve their quadratic ***(M1)*** $\frac{1}{4}x(x+8) = 0$ OR $x(x+8) = 0$ $x = -8$ ***A1*** **Note:** Accept both solutions $x = -8$ and $x = 0$ here, $x = -8$ may be seen in working to find coordinates of Q or distance. substituting their value of x (not $x = 0$) into their $f(x)$ or their L ***(M1)*** $y = -(-8) - 4$ OR $y = \frac{1}{4}(-8+2)^2 - 5$ $Q(-8,\ 4)$ ***A1*** correct substitution into distance formula ***(A1)*** $\sqrt{(-8-0)^2 + (4-(-4))^2}$ $distance = \sqrt{128}$ $\left(= 8\sqrt{2}\right)$ ***A1*** ***[8 marks]***

15. 22M.1.SL.TZ1.6

(a)

Describe these two transformations.

[2]

Markscheme

translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction ***A1***
translation (shift) by q upwards/positive vertical direction ***A1***

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

(b)

The y-intercept of the graph of g is at $(0,\ r)$.

Given that $g(x) \geq 7$, find the smallest value of r.

[5]

Markscheme

METHOD 1

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) ***(M1)***

$$-4 + 2.5 + q \geq 7$$

$q \geq 8.5$ (accept $q = 8.5$) ***A1***

substituting $x = 0$ and their q $(= 8.5)$ to find r ***(M1)***

$$(r =)\ 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + 8.5$$

$4 + 2.5 + 8.5$ ***(A1)***

smallest value of r is 15 ***A1***

METHOD 2

substituting $x = 0$ to find an expression (for r) in terms of q ***(M1)***

$$(g(0) = r =)\ 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + q$$

$(r =)\ 6.5 + q$ ***A1***

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 ***(M1)***

$$-4 + 2.5 + q \geq 7$$

$-4 + 2.5 + (r - 6.5) \geq 7$ (accept $=$) ***(A1)***

smallest value of r is 15 ***A1***

METHOD 3

$4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q = 4\cos x + 2.5 + q$ ***A1***

y-intercept of $4\cos x + 2.5 + q$ is a maximum ***(M1)***

amplitude of $g(x)$ is 4 ***(A1)***

attempt to find least maximum ***(M1)***

$$r = 2 \times 4 + 7$$

smallest value of r is 15 ***A1***

[5 marks]

16. 22M.1.SL.TZ1.7

(a)

Find the value of q.

[2]

Markscheme
EITHER attempt to use $x = -\frac{b}{2a}$ ***(M1)*** $q = -\frac{-12}{2 \times 3}$ **OR** attempt to complete the square ***(M1)*** $3(x-2)^2 - 12 + p$ **OR** attempt to differentiate and equate to 0 ***(M1)*** $f''(x) = 6x - 12 = 0$ **THEN** $q = 2$ ***A1*** ***[2 marks]***

(b.i)

Write down the value of the discriminant of f'.

[1]

Markscheme
discriminant $= 0$ ***A1*** ***[1 mark]***

(b.ii)

Hence or otherwise, find the value of p.

[3]

Markscheme

EITHER
attempt to substitute into $b^2 - 4ac$ ***(M1)***
$(-12)^2 - 4 \times 3 \times p = 0$ ***A1***

OR
$f'(2) = 0$ ***(M1)***
$-12 + p = 0$ ***A1***

THEN
$p = 12$ ***A1***

[3 marks]

(c)

Find the value of the gradient of the graph of f' at $x = 0$.

[3]

Markscheme

$f''(x) = 6x - 12$ ***A1***
attempt to find $f''(0)$ ***(M1)***
$= 6 \times 0 - 12$
gradient $= -12$ ***A1***

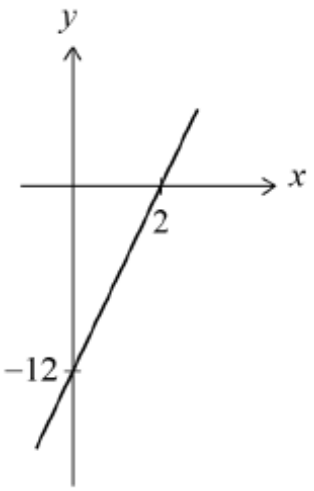
[3 marks]

(d)

Sketch the graph of f'', the second derivative of f. Indicate clearly the x-intercept and the y-intercept.

[2]

Markscheme



A1A1

Note: Award ***A1*** for line with positive gradient, ***A1*** for correct intercepts.

[2 marks]

(e.i)

Write down the value of a.

[1]

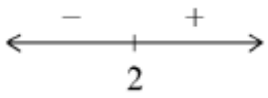
Markscheme
$a = 2$ ***A1***
[1 mark]

(e.ii)

Find the values of x for which the graph of f is concave-down. Justify your answer.

[2]

Markscheme
$x < 2$ ***A1***
$f''(x) < 0$ (for $x < 2$) OR the f'' is below the x-axis (for $x < 2$)



OR f'' (sign diagram must be labelled f'') ***R1***

[2 marks]

17. 22M.1.SL.TZ2.6

(a)

Show that $b = 21$.

[2]

Markscheme

EITHER
recognises the required term (or coefficient) in the expansion ***(M1)***
$bx^5 = x^5 1^2$ OR $b =$ OR

$$b = \frac{7!}{2!\,5!}\left(= \frac{7!}{2!\,(7-2)!}\right)$$

correct working ***A1***
$\frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1}$ OR $\frac{7\times6}{2!}$ OR $\frac{42}{2}$

OR
lists terms from row 7 of Pascal's triangle ***(M1)***
$1,\ 7,\ 21, \ldots$ ***A1***

THEN
$b = 21$ ***AG***

[2 marks]

(b)

The third term in the expansion is the mean of the second term and the fourth term in the expansion.

Find the possible values of x.

[5]

Markscheme

$a = 7$ ***(A1)***
correct equation ***A1***
$21x^5 = \frac{ax^6+35x^4}{2}$ OR $21x^5 = \frac{7x^6+35x^4}{2}$
correct quadratic equation ***A1***
$7x^2 - 42x + 35 = 0$ OR $x^2 - 6x + 5 = 0$ (or equivalent)
valid attempt to solve **their** quadratic ***(M1)***
$(x-1)(x-5) = 0$ OR $x = \frac{6 \pm \sqrt{(-6)^2-4(1)(5)}}{2(1)}$
$x = 1,\ x = 5$ ***A1***

Note: Award final ***A0*** for obtaining $x = 0,\ x = 1,\ x = 5$.

[5 marks]

18. 22M.1.SL.TZ2.7

(a)

Write down the equation of the axis of symmetry.

[1]

Markscheme

$x = 3$ ***A1***

Note: Must be an equation in the form " $x =$ ". Do not accept 3 or $\frac{-b}{2a} = 3$.

[1 mark]

(b.i)

Write down the values of h and k.

[2]

Markscheme

$h = 3,\ k = 4$ (accept $a(x-3)^2 + 4$) ***A1A1***

[2 marks]

(b.ii)

Point Q has coordinates $(5,\ 12)$. Find the value of a.

[2]

Markscheme

attempt to substitute coordinates of Q ***(M1)***

$12 = a(5-3)^2 + 4,\ \ 4a + 4 = 12$

$a = 2$ ***A1***

[2 marks]

(c)

Find the equation of L.

[4]

Markscheme

recognize need to find derivative of f ***(M1)***

$f'(x) = 4(x-3)$ or $f'(x) = 4x - 12$ ***A1***

$f'(5) = 8$ (may be seen as gradient in their equation) ***(A1)***

$y - 12 = 8(x-5)$ or $y = 8x - 28$ ***A1***

Note: Award ***A0*** for $L = 8x - 28$.

[4 marks]

(d)

Find the values of d for which g is an increasing function.

[3]

Markscheme

METHOD 1
Recognizing that for g to be increasing, $f(x) - d > 0$, or $g' > 0$ ***(M1)***
The vertex must be above the x-axis, $4 - d > 0,\ d - 4 < 0$ ***(R1)***
$d < 4$ ***A1***

METHOD 2
attempting to find discriminant of g' ***(M1)***
$(-12)^2 - 4(2)(22 - d)$
recognizing discriminant must be negative ***(R1)***
$-32 + 8d < 0$ OR $\Delta < 0$
$d < 4$ ***A1***

[3 marks]

(e)

Find the values of x for which the graph of g is concave-up.

[3]

Markscheme

recognizing that for g to be concave up, $g'' > 0$ ***(M1)***
$g'' > 0$ when $f' > 0,\ 4x - 12 > 0,\ x - 3 > 0$ ***(R1)***
$x > 3$ ***A1***

[3 marks]

19. 22N.1.SL.TZ0.6

(a)

Determine the value of $\mathrm{P}(A \cap B)$ in the case where the events A and B are independent.

[1]

Markscheme

$\mathrm{P}(A \cap B) = 0.24$ ***A1***

[1 mark]

(b)

Determine the minimum possible value of $\mathrm{P}(A \cap B)$.

[3]

Markscheme
$\mathrm{P}(A \cup B) = 1.1 - \mathrm{P}(A \cap B)$ ***(A1)*** $(0 \leq)\mathrm{P}(A \cup B) \leq 1$ ***(M1)*** **Note:** This may be conveyed in a clearly labelled diagram or written explanation where $\mathrm{P}(A \cup B) = 1$ the minimum value of $\mathrm{P}(A \cap B)$ is 0.1 ***A1*** ***[3 marks]***

(c)

Determine the maximum possible value of $\mathrm{P}(A \cap B)$, justifying your answer.

[2]

Markscheme
A is a subset of B (so $\mathrm{P}(A \cap B) = \mathrm{P}(A)$). ***R1*** **Note:** This may be conveyed in a clearly labelled diagram where A is completely inside B, or in a written explanation indicating that $\mathrm{P}(A \cap B) = \mathrm{P}(A)$ so the maximum value of $\mathrm{P}(A \cap B)$ is 0.3 ***A1*** **Note:** Do not award ***R0A1***. ***[2 marks]***

20. 22N.1.SL.TZ0.7

(a)

The graph of a quadratic function f has its vertex at the point $(3,\ 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f(x) = a(x-h)^2 + k$.

[3]

Markscheme

correct substitution of $h = 3$ and $k = 2$ into $f(x)$ ***(A1)***

$$f(x) = a(x-3)^2 + 2$$

correct substitution of $(5,\ 0)$ ***(A1)***

$$0 = a(5-3)^2 + 2 \quad \left(a = -\frac{1}{2}\right)$$

Note: The first two A marks are independent.

$f(x) = -\frac{1}{2}(x-3)^2 + 2$ ***A1***

[3 marks]

(b.i)

find the value of p and the value of t.

[4]

Markscheme

METHOD 1

correct substitution of $(1,\ 4)$ ***(A1)***

$$p + (t-1) - p = 4$$

$t = 5$ ***A1***

substituting their value of t into $9p - 3(t-1) - p = 4$ ***(M1)***

$$8p - 12 = 4$$

$p = 2$ ***A1***

METHOD 2

correct substitution of ONE of the coordinates $(-3,\ 4)$ or $(1,\ 4)$ ***(A1)***

$9p - 3(t-1) - p = 4$ OR $p + (t-1) - p = 4$

valid attempt to solve their two equations ***(M1)***

$p = 2,\ t = 5$ ***A1A1***

$(g(x) = 2x^2 + 4x - 2)$

[4 marks]

(b.ii)

find the range of g.

[3]

Markscheme

attempt to find the x-coordinate of the vertex ***(M1)***
$x = \frac{-3+1}{2} (= -1)$ OR $\frac{-4}{2\times 2}$ OR $4x + 4 = 0$ OR $2(x+1)^2 - 4$
y-coordinate of the vertex $= -4$ ***(A1)***
correct range ***A1***
$[-4, +\infty[$ OR $y \geq -4$ OR $g \geq -4$ OR $[-4, \infty)$

[3 marks]

(c)

The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}$, $p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t-1)x - p$ have two distinct points of intersection for every possible value of p and t.

[6]

Markscheme

equating the two functions or equations ***(M1)***
$g(x) = j(x)$ OR $px^2 + (t-1)x - p = -x + 3p$
$px^2 + tx - 4p = 0$ ***(A1)***
attempt to find discriminant (do not accept only in quadratic formula) ***(M1)***
$\Delta = t^2 + 16p^2$ ***A1***
$\Delta = t^2 + 16p^2 > 0$, because $t^2 \geq 0$ and $p^2 > 0$, therefore the sum will be positive ***R1R1***

Note: Award ***R1*** for recognising that Δ is positive and ***R1*** for the reason.

There are two distinct points of intersection between the graphs of g and j. ***AG***

[6 marks]

21. 25M.2.SL.TZ1.6

(a)

Find the gradient of T_k in terms of k.

[3]

Markscheme
recognition of the need to differentiate ***(M1)***
$h'(x) = \frac{\pi}{50}\left(-15\sin\left(\frac{\pi x}{50}\right)\right)\left(= -\frac{15\pi}{50}\sin\left(\frac{\pi x}{50}\right) = -\frac{3\pi}{10}\sin\left(\frac{\pi x}{50}\right)\right)$ ***A1A1***
$h'(k) = -\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right)\left(= -\frac{3\pi}{10}\sin\left(\frac{\pi k}{50}\right)\right)$
Note: Award ***A1*** for $-15\sin\left(\frac{\pi k}{50}\right)$ and ***A1*** for factor of $\frac{\pi}{50}$. Award ***A1A0*** for a correct expression with additional terms or additional factors. ***[3 marks]***

(b)

Find the possible values of k.

[3]

Markscheme
recognition that gradient of tangent $= -\tan\left(\frac{\pi}{8}\right)$ **OR** $\tan\left(\frac{7\pi}{8}\right)$ ***(M1)***
Note: Accept $\tan\left(\frac{\pi}{8}\right)$ **OR** $-\tan\left(\frac{7\pi}{8}\right)$ for the ***(M1)***
setting their $h'(k)$ equal to $-\tan\left(\frac{\pi}{8}\right)$ **OR** $\tan\left(\frac{7\pi}{8}\right)$ $(= -0.414213\ldots)$ ***(A1)***
$-\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = -\tan\left(\frac{\pi}{8}\right)$ **OR** $-\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = \tan\left(\frac{7\pi}{8}\right)$
$k = 7.24211\ldots$, $k = 42.7578\ldots$ $k = 7.24$, $k = 42.8$ ***A1***

[3 marks]

22. 25M.2.SL.TZ1.7

(a)

Show that the probability that Lynn scores 5 points in one round is $\frac{5}{18}$.

[2]

Markscheme

P (scores 5 points) $= \frac{10}{36}$ ***A1A1*** **Note:** Award ***A1*** for numerator of 10 and ***A1*** for denominator of 36.

$= \frac{5}{18}$ ***AG***

[2 marks]

(b)

Find the probability that Lynn scores no points in one round.

[2]

Markscheme

METHOD 1 attempts to list or count all outcomes with a difference of more than one ***(M1)*** $4 + 3 + 3 + 3 + 3 + 4 (= 20)$ P (scores 0 points) $= \frac{20}{36}\left(= \frac{5}{9} = 0.555555\ldots = 0.556\right)$ ***A1*** **METHOD 2** attempts to subtract P (scores 5 points) and P (scores 10 points) from 1 ***(M1)***

P (scores 0 points) $= 1 - \frac{10}{36} - \frac{6}{36} = \frac{20}{36}\left(= \frac{5}{9} = 0.555555\ldots = 0.556\right)$ ***A1***

[2 marks]

(c)

Find E (X).

[4]

Markscheme

$P(\text{scores } 10 \text{ points}) = \frac{6}{36}\left(=\frac{1}{6}\right)$ (may be seen in a table) ***(A1)***

x	0	5	10
$\mathrm{P}(X=x)$	$\frac{20}{36}$	$\frac{10}{36}$	$\frac{6}{36}$

attempt to use formula for expected value ***(M1)*** $E(X) = \left(0 \times \frac{20}{36} + \right) 5 \times \frac{10}{36} + 10 \times \frac{6}{36}$ ***(A1)*** $= \frac{110}{36}\left(= \frac{55}{18} = 3.05555\ldots = 3.06\right)$ ***A1***

[4 marks]

(d)

Hence, estimate the total number of points that Lynn scores if she plays 90 rounds.

[2]

Markscheme

recognition that their part c) must be multiplied by 90 ***(M1)*** $\frac{110}{36} \times 90 = 275$ ***A1***

[2 marks]

(e)

Find the probability that Lynn wins a prize.

[4]

Markscheme

recognise that round scores must be five 10s or four 10s and a 5 ***(M1)***

P (five 10s) $= \left(\frac{1}{6}\right)^5$, P (four 10s and a 5) $= 5\left(\frac{1}{6}\right)^4\left(\frac{5}{18}\right)$ ***(A1) (A1)***

P (Lynn wins a prize) $= 0.000128601\ldots + 0.00107167\ldots$

$= 0.00120027\ldots (= 1.20027\ldots \times 10^{-3})$

$= 0.00120 \left(= 1.20 \times 10^{-3} = \frac{7}{5\,832}\right)$ (accept 0.0012) ***A1***

[4 marks]

23. 24N.2.SL.TZ1.6

[5]

Markscheme

METHOD 1 recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1*** (discriminant =) $(-r)^2 - 4(3)(r-2)$ $(= r^2 - 12r + 24)$ (seen anywhere) ***(A1)*** $2.53589\ldots$ $\left(= 6 - 2\sqrt{3}\right)$ AND $9.46410\ldots$ $\left(= 6 + 2\sqrt{3}\right)$ (seen anywhere) ***(A1)***
recognition that discriminant of $3x^2 - rx + r - 2$ is less than zero ***(M1)***
$2.54 < r < 9.46$ $\left(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3}\right)$ ***A1*** **Note**: Accept $2.54 \leq r \leq 9.46$. **METHOD 2** recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) ***R1*** **EITHER** minimum when $x = \frac{r}{6} \Rightarrow$
$(y =)\ 3\left(\frac{r}{6}\right)^2 - r\left(\frac{r}{6}\right) + r - 2$ (> 0) ***(A1)***
attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)*** **OR** $x < 1 \Rightarrow r > \frac{3x^2-2}{x-1}$ OR $x > 1 \Rightarrow r < \frac{3x^2-2}{x-1}$ ***(A1)***
attempt to find local minumum AND local maximum of $r = \frac{3x^2-2}{x-1}$ ***(M1)***
THEN $(r >)\ 2.53589\ldots$ $\left(= 6 - 2\sqrt{3}\right)$ AND $(r <)\ 9.46410\ldots$ $\left(= 6 + 2\sqrt{3}\right)$ (seen anywhere) ***(A1)***
$2.54 < r < 9.46$ $\left(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3}\right)$ ***A1*** **Note**: Accept $2.54 \leq r \leq 9.46$.
[5 marks]

24. 24N.2.SL.TZ1.7

[[N/A]]

(a)

Find the percentage of *MyLife* users aged 55 years or older.

[1]

Markscheme
$16.7037 \ldots 16.7\ (\%)$ ***A1*** ***[1 mark]***

(b)

A sample of 1000 *MyLife* users is chosen at random. Find the probability that fewer than 150 of them are aged 55 years or older.

[3]

Markscheme
recognition of binomial ***(M1)*** $X \sim B(1000,\ 0.167 \ldots)$ attempt to find $P(X < 150)$ OR $P(X \leq 149)$ ***(M1)*** $0.0669378 \ldots P(X < 150) = 0.0669$ (accept 0.0673 from previous 3sf) ***A1*** ***[3 marks]***

(c)

Given that a *MyLife* user chosen at random is 55 years or older, find the probability that they are 75 years or older.

[4]

Markscheme
recognition of conditional probability in context ***(M1)*** $P(\text{age} \geq 75 \mid \text{age} \geq 55)$ OR $\frac{P(\text{age} \geq 55 \cap \text{age} \geq 75)}{P(\text{age} \geq 55)}$ **Note**: Recognition must be shown in context either in words or symbols, not just $P(A \mid B)$. $\frac{P(\text{age} \geq 75)}{P(\text{age} \geq 55)}$ ***(A1)*** $\frac{2.1}{(2.1+12.9)}$ OR $\frac{0.0233853\ldots}{0.167}$ ***(A1)*** 0.14 (exact) $\left(= \frac{7}{50}\right)$ ***A1*** **Note**: Condone use of $"a"$ or $"X"$ or any letter for age. ***[4 marks]***

(d)

List the mid-interval value for each class interval.

[1]

Markscheme
$15,\ 26.5,\ 45,\ 65,\ 85$ ***A1*** ***[1 mark]***

(e)

Hence, for *MyLife* users, estimate

[[N/A]]

(e.i)

the mean age;

[1]

Markscheme
38.0740 ... mean $= 38.1$ ***A1*** ***[1 mark]***

(e.ii)

the variance of the ages.

[2]

Markscheme
variance is square of standard deviation ***(M1)*** $16.3639 \ldots^2$ $267.778 \ldots$ variance$= 268$ ***A1*** ***[2 marks]***

(f.i)

Identify the diagram which best represents the age distributions for the users of *MyLife* and *SmallTalk*.

[1]

Markscheme
Graph D ***A1*** **Note**: Allow ***FT*** from incorrect values for mean and variance in (e). ***[1 mark]***

(f.ii)

In your chosen diagram, identify which social media platform is represented by the dotted line.

[1]

Markscheme
SmallTalk ***A1*** ***[1 mark]***

25. 25M.2.SL.TZ3.6

(a)

Find an expression for $f'(x)$, in terms of a.

[3]

Markscheme
use of quotient rule or product rule ***(M1)*** **EITHER** $f'(x) = \frac{3\times2\times(2x+a)^2(x+5)^2-2(2x+a)^3(x+5)}{((x+5)^2)^2}$ ***(A1)*** $f'(x) = \frac{6(2x+a)^2(x+5)^2-2(2x+a)^3(x+5)}{(x+5)^4}\left(=\frac{2(x-a+15)(2x+a)^2}{(x+5)^3}\right)$ ***A1*** **OR** $f'(x) = 3\times2\times(2x+a)^2(x+5)^{-2}-2(2x+a)^3(x+5)^{-3}$ ***(A1)***

$f'(x) = 6(2x+a)^2(x+5)^{-2} - 2(2x+a)^3(x+5)^{-3} (= 2(x-a+15)(2x+a)^2(x+5)^{-3})$ ***A1***

[3 marks]

(b)

Find the two possible values of a.

[4]

Markscheme

recognizing $f'(1)$ is equal to $\tan 70° (= 2.74747)$ ***(M1)***

$\frac{6(2+a)^2 6^2 - 2(2+a)^3 \times 6}{6^4} = \tan 70°$ **OR** $6(2+a)^2 6^{-2} - 2(2+a)^3 \times 6^{-3} = \tan 70°$ ***(A1)*** $2.72844...$ and $14.96968...$ $a = 2.73,\ 15.0$ ***A1A1***

[4 marks]

26. 25M.2.SL.TZ3.7

(a.i)

Write down the amplitude of f.

[1]

Markscheme

$0.636619...$ (amplitude $=$) 0.637 $\left(= \frac{2}{\pi}\right)$ ***A1***

[1 mark]

(a.ii)

Find the period of f.

[2]

Markscheme

$\frac{2\pi}{3\pi}$ **OR** 3 complete waves every 2 radians ***(A1)*** (period $=$) 0.667 $\left(=\frac{2}{3}\right)$ ***A1***

[2 marks]

(b)

The point P has coordinates $(1.63,\ 2.16)$. State whether P lies above, below or on the graph of f. Justify your answer.

[3]

Markscheme
substituting to find $f(1.63)$ ***(M1)*** $(f(1.63) =)\ 2.21564\ldots$ ***A1*** $2.21564\ldots > 2.16 \Rightarrow$ lies below (the graph of f) ***R1*** ***[3 marks]***

(c)

Write down the gradient of the line L_1.

[1]

Markscheme
$(m_{L_1} =)\ 0.167\ \left(=\frac{1}{6}\right)$ ***A1*** ***[1 mark]***

(d.i)

Find the gradient of L_2.

[2]

Markscheme
substituting their m_{L_1} into $-\frac{1}{m_{L_1}}$ ***(M1)*** $(m_{L_2} =) -6$ ***A1***

[2 marks]

(d.ii)

Hence, or otherwise, find the equation of L_2.

[1]

Markscheme
$y-2=-6(x-1)$ **OR** $y=-6x+8$ **OR** $6x+y-8=0$ ***A1*** ***[1 mark]***

(e)

Find the coordinates of B.

[2]

Markscheme
equating L_1 and f ***(M1)*** $B\,(1.64856\ldots,\ 2.10809\ldots)$ $B\,(1.65,\ 2.11)$ ***A1*** ***[2 marks]***

(f)

Find the area of the shaded region.

[3]

Markscheme
EITHER Attempt to form the required integral involving subtraction in either order. ***(M1)*** $\int_{t_1}^{t_2}\lvert L_1-f(x)\rvert\ dx$ correct expression with limits 1 and *their* x value at B $\int_1^{1.64856\ldots}\left\lvert\left(\frac{1}{6}x+\frac{11}{6}\right)-\left(\frac{2}{\pi}\sin 3\pi x+2\right)\right\rvert\ dx$ (or equivalent) ***(A1)*** **OR** intersection point between A and B is $(1.34287\ldots,\ 2.05714\ldots)$ subtracting L_1 and f, in either order, and substituting into area of region formula for one area with correct

limits. ***(M1)***

Two correct expressions **OR** correct values ***(A1)*** $\int_1^{1.34287\ldots}\left(\left(\frac{1}{6}x+\frac{11}{6}\right)-\left(\frac{2}{\pi}\sin 3\pi x+2\right)\right)dx=(0.144618\ldots)$ **AND**

$\int_{1.34287\ldots}^{1.64856\ldots}\left(\left(\frac{2}{\pi}\sin 3\pi x+2\right)-\left(\frac{1}{6}x+\frac{11}{6}\right)\right)dx=(0.108585\ldots)$ **THEN**
$0.253204\ldots$ (area $=$) 0.253 ***A1***
[3 marks]

27. 25M.2.SL.TZ2.6

[6]

Markscheme

METHOD 1 recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}}=\frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)*** $\text{area}_{\text{triangle}}=21.3333\ldots$ **OR** $\text{area}_{\text{sector}}=34.1333\ldots$ ***A1***
correct equation in r and θ ***(A1)***

$\frac{1}{2}r^2\theta=34.1333\ldots$ **OR** $\frac{1}{2}r^2\sin\theta=21.3333\ldots$ **OR** $\frac{1}{2}r^2(\theta-\sin\theta)=12.8$ (seen anywhere)

correct equation in one variable ***A1*** $\frac{1}{2}r^2\left(\frac{68.2666\ldots}{r^2}-\sin\left(\frac{68.2666..}{r^2}\right)\right)=12.8$ **OR** $\frac{1}{2}\left(\frac{68.2666..}{\theta}\right)\sin\theta=21.3333\ldots$ **OR** $\frac{1}{2}\left(\frac{68.2666..}{\theta}\right)(\theta-\sin\theta)=12.8$
attempt to solve their equation or use of graph ***(M1)*** $\theta=1.59934\ldots$ $6.53330\ldots$ $r=6.53$ ***A1*** **METHOD 2** recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}}=\frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)*** $\text{area}_{\text{segment}}=\frac{1}{2}r^2(\theta-\sin\theta)$ (seen anywhere) ***(A1)*** $\frac{\frac{1}{2}r^2(\theta-\sin\theta)}{\frac{1}{2}r^2\sin\theta}=\frac{3}{5}$ correct equation without r ***A1***
$\frac{(\theta-\sin\theta)}{\sin\theta}=\frac{3}{5}$ $\theta=1.59934\ldots$ ***(A1)***
attempt to solve for r using their θ ***(M1)*** $\frac{1}{2}r^2(1.59\ldots-\sin 1.59\ldots)=12.8$ $6.53330\ldots$ $r=6.53$ ***A1*** **METHOD 3** recognising $\text{area}_{\text{segment}}=\frac{3}{8}\times\text{area}_{\text{sector}}$ (seen anywhere) ***(M1)*** $\text{area}_{\text{segment}}=\frac{1}{2}r^2(\theta-\sin\theta)$ (seen anywhere) ***(A1)*** $\frac{1}{2}r^2(\theta-\sin\theta)=\frac{3}{8}\times\frac{1}{2}r^2\theta$ correct equation without r ***A1*** $\frac{1}{2}(\theta-\sin\theta)=\frac{3}{8}\times\frac{1}{2}\theta$ $\theta=1.59934\ldots$ ***(A1)***

attempt to solve for r using their θ ***(M1)*** $\frac{1}{2}r^2(1.59\ldots - \sin 1.59\ldots) = 12.8$ $6.53330\ldots$ $r = 6.53$ ***A1***
[6 marks]

28. 25M.2.SL.TZ2.7

[[N/A]]

(a)

Find JL.

[4]

Markscheme

$J\hat{C}L = 28°$ ***(A1)*** attempt to substitute into cosine rule ***(M1)*** $JL^2 = (5.5)^2 + 8^2 - 2\,(5.5)\,(8)\cos 28°$ ***(A1)*** $4.068244\ldots$ $JL = 4.07$ (km) ***A1*** **Note:** Award ***(A1)(M1)(A1)A0*** for an answer of 13.4 $(13.3775\ldots)$ (km) for working in radians.
[4 marks]

(b)

Find BL.

[3]

Markscheme

valid attempt to solve ***(M1)*** use a trigonometric ratio or sine rule in triangle BLX

$\tan 0.94° = \frac{60}{BL}$ **OR** $\tan 89.06° = \frac{BL}{60}$ **OR** $\frac{BL}{\sin 89.06°} = \frac{60}{\sin 0.94°}$ ***(A1)***

$3\,656.849288\ldots$ **OR** $3.656849\ldots$ $BL = 3\,660$ (m) **OR** $BL = 3.66$ (km) ***A1***
[3 marks]

(c)

Find the bearing from L to B.

[2]

Markscheme
valid approach ***(M1)*** clear diagram **OR** $180 + 121$ **OR** $360 - (180 - 121)$ **OR** $270 + (121 - 90)$ $301°$ (accept $N59°W$) ***A1*** ***[2 marks]***

(d)

Find the time, in minutes, for the lighthouse rescue boat to reach the Jet Ski at B.

[2]

Markscheme
$\frac{t}{3.656\ldots} = \frac{60}{48}$ **OR** $t = \frac{3.65684\ldots}{48}$ $(= 0.0761843\ldots)$ ***(A1)*** $4.57106\ldots$ $t =$ 4.57 (min) ***A1*** ***[2 marks]***

(e)

Determine which rescue boat reaches the Jet Ski first. Justify your answer.

[4]

Markscheme
attempt to use cosine rule to find distance BC ***(M1)*** $(3.656\ldots)^2 + 8^2 - 2(3.656\ldots)(8)\cos 31°$ $(= 27.2200\ldots)$ $BC = 5.21728\ldots$ ***(A1)*** $\frac{t}{5.21728\ldots} = \frac{60}{55}$ **OR** $t = \frac{5.21728\ldots}{60}$ $(= 0.094859\ldots)$ time for coast guard rescue boat $=$ $5.69158\ldots$ (min) **OR** $0.094859\ldots$ (hour) ***A1*** $5.69158\ldots >$ $4.57106\ldots$ (min) **OR** $0.094859\ldots > 0.076184\ldots$ (hour) the lighthouse rescue boat will reach the jet-ski first. ***A1***

[4 marks]

29. 24M.2.SL.TZ1.7

(a)

Show that the inverse function is given by $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$.

[3]

Markscheme

METHOD 1 attempt to interchange x and y ***M1*** **Note:** This ***M1*** may be awarded at any stage in the working. attempt to rearrange using definition of natural log or take the natural log of both sides ***M1*** $\frac{2x}{3} = e^{y-2} \Rightarrow \ln\left(\frac{2x}{3}\right) = y - 2$ OR $x = \frac{3}{2}e^{y-2} \Rightarrow \ln(x) = \ln\left(\frac{3}{2}\right) + y - 2$ ***A1*** $y = 2 + \ln\left(\frac{2x}{3}\right)$ so $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$ ***AG*** **METHOD 2** attempt to verify that $(f \circ f^{-1})(x) = x$ ***M1*** $(f \circ f^{-1})(x) = \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)+2-2}\left(= \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)}\right)$ attempt to use definition of natural log ***M1*** $(f \circ f^{-1})(x) = \frac{3}{2} \times \frac{2x}{3}$ ***A1*** $(f \circ f^{-1})(x) = x$ ***AG***
[3 marks]

(b)

Find PQ.

[3]

Markscheme

$(0.264456\ldots, 0.264456\ldots)$ AND $(2.51799\ldots, 2.51799\ldots)$ ***(A1)*** **Note:** Award ***A1*** for $0.264456\ldots$ and $2.51799\ldots$ seen. attempt to put their values in distance formula or use of the isosceles right-angled triangle ***(M1)*** $\sqrt{(2.51799\ldots - 0.264456\ldots)^2 + (2.51799\ldots - 0.264456\ldots)^2}$ OR $\sqrt{2} \times (2.51799\ldots - 0.264456\ldots) = 3.18689\ldots = 3.19$ ***A1***
[3 marks]

(c)

Write down

[[N/A]]

(c.i)

an expression for $g(x)$;

[2]

Markscheme
$g(x) = -\frac{3}{2}e^{x-2} + 5$ OR $g(x) = -f(x) + 5$ ***A1A1*** **Note:** Award ***A1*** for each correct term. ***[2 marks]***

(c.ii)

the domain of g.

[1]

Markscheme
$0 \leq x \leq 4$ ***A1*** ***[1 mark]***

(d)

Solve the equation $f(x) = g(x)$. Give your answer in the form $x = a + \ln b$, where $a,\ b\ \in \mathbb{Q}$.

[3]

Markscheme
$\frac{3}{2}e^{x-2} = -\frac{3}{2}e^{x-2} + 5$ OR $f(x) = -f(x) + 5$ attempt to collect together terms in e^{x-2} or $f(x)$ ***(M1)*** $3e^{x-2}$ OR $2f(x) = 5\ e^{x-2} = \frac{5}{3}$ OR $x =$

$f^{-1}\left(\frac{5}{2}\right)$ ***(A1)*** $x = 2 + \ln\left(\frac{5}{3}\right)$ ***A1*** $\left(a = 2,\ b = \frac{5}{3}\right)$ **Note:** Award ***A1*** for each correct term given in exact form.
[3 marks]

30. 24M.2.SL.TZ2.6

[5]

Markscheme

METHOD 1 correct inequality or equation involving $P(X = 0)$ ***(A1)*** $1 - P(X = 0) > 0.99$ OR $P(X = 0) < 0.01$ OR $1 - P(X = 0) = 0.99$ OR $P(X = 0) = 0.01$ attempts to solve their inequality (equality) involving 0.75^n for n ***(M1)*** $1 - 0.75^n > 0.99$ OR $0.75^n < 0.01$ OR $0.75^n = 0.01$ OR $1 - 0.75^n = 0.99$ **Note:** Valid solving attempts include graphical, use of logarithms, tabular or trial and error. **EITHER** $n > 16.0078\ldots$ OR $n = 16.0078\ldots$ ***(A2)*** the least value of n is 17 ***A1*** **OR** $P(X = 0) = 0.010022\ldots\ (> 0.01)$ (corresponding to $n = 16$) ***(A1)*** $P(X = 0) = 0.0075169\ldots\ (< 0.01)$ ***(A1)*** corresponding to $n = 17$ (which is the least value of n) ***A1*** **METHOD 2 (TABLE ONLY APPROACH)** attempts to use binomial cdf to calculate a correct value of $P(X \geq 1)$ for one value of n ***(M1)*** calculates correct values of $P(X \geq 1)$ for at least one value of n ***(A1)*** $P(X \geq 1) = 0.989977\ldots\ (< 0.99)$ (corresponding to $n = 16$) ***(A1)*** $P(X \geq 1) = 0.992483\ldots\ (> 0.99)$ ***(A1)*** corresponding to $n = 17$ (which is the least value of n) ***A1***
[5 marks]

31. 24M.2.SL.TZ2.7

(a)

Write down the value of k.

[2]

Markscheme

recognition to add μ and σ ***(M1)*** 49.8 (cm) ***A1***
[2 marks]

(b)

Find the probability that a randomly selected carp is greater than $48\,cm$ in length.

[2]

Markscheme
$P(L > 48)$ ***(M1)*** **Note:** Award ***(M1)*** for a clearly labelled diagram. $= 0.283854\ldots = 0.284$ ***A1*** ***[2 marks]***

(c)

It is known that $99\,\%$ of carp in the lake have a length greater than $x\,cm$. Find the value of x.

[2]

Markscheme
$P(L > x) = 0.99$ OR $P(L < x) = 0.01$ ***(M1)*** **Note:** Award ***(M1)*** for a clearly labelled diagram or the use of inverse normal. $x = 35.8293\ldots\ x = 35.8$ (cm) ***A1*** ***[2 marks]***

(d)

Consider a random sample of 100 carp from the lake.

[[N/A]]

(d.i)

Find the expected number of carp with lengths between $40\,cm$ and $56\,cm$.

[3]

Markscheme

$P(40 < L < 56) = 0.902149$ ... (may be seen in part (ii)) ***(A1)*** attempts to find $100 \times P(40 < L < 56)$ with their probability ***(M1)*** $= 90.2149$ ... $= 90.2$ ***A1*** **Note:** Accept 90 or 91.
[3 marks]

(d.ii)

Find the probability that in this sample, exactly 95 carp have a length between $40\ cm$ and $56\ cm$.

[2]

Markscheme

recognizes binomial distribution ***(M1)*** $X \sim B(100,\ 0.902149\ldots)$
$P(X = 95) = 0.038105\ldots = 0.0381$ ***A1***
[2 marks]

(e)

Find the probability that a randomly selected carp has a length recorded as $45.6\ cm$.

[3]

Markscheme

$P(45.55 \leq L < 45.7)$ ***(M1)(A1)*** **Note:** Award ***(M1)*** for any reasonable interval centred on 45.6, no wider than $P(45.5 \leq L < 45.7)$. Accept either of $P(45.55 \leq L \leq 45.65)$ or $P(45.55 < L < 45.65)$. $= 0.009498\ldots = 0.00950$ ***A1***
[3 marks]

32. 23N.2.SL.TZ2.6

[5]

Markscheme

$E(X) = k + 2k^2 + 3a + 4k^3 = 2.3$ ***(A1)*** $k + k^2 + a + k^3 = 1$ ***(A1)*** **Note:** The first two ***A*** marks are independent of each other. **EITHER** (finding intersections of functions) attempt to make a the subject in both of their equations ***(M1)*** $a = 1 - k - k^2 - k^3$ and $a = \frac{1}{3}(2.3 - k - 2k^2 - 4k^3)$ use of graph or table to attempt to find intersection ***(M1)*** **OR** (solving algebraically) attempt to solve their equations algebraically to find a cubic in k ***(M1)*** $k^3 - k^2 - 2k + 0.7 = 0$ OR $3(1 - k - k^2 - k^3) = 2.3 - k - 2k^2 - 4k^3$ (or equivalent) attempt to solve their cubic in k ***(M1)*** **THEN** $a = 0.552839\ldots$ OR $k = 0.315870\ldots$ (other solutions to cubic are $k = -1.18538\ldots, 1.86951\ldots$) $a = 0.553$ ***A1***

[5 marks]

33. 23N.2.SL.TZ2.7

[[N/A]]

(a)

Show that the total surface area, $S\ cm^2$, of the solid is given by $S = \frac{90}{x} + 4\pi x^2$.

[3]

Markscheme

$\pi x^2 h = 45$ ***(A1)*** attempt to rearrange AND substitute their h into the expression for the total surface area ***(M1)*** $S = 2\pi x\left(\frac{45}{\pi x^2}\right) + 4\pi x^2$ ***A1*** $S = \frac{90}{x} + 4\pi x^2$ ***AG***

[3 marks]

(b)

[N/A]

[[N/A]]

(b.i)

Find an expression for $\frac{dS}{dx}$.

[2]

Markscheme
$\frac{dS}{dx} = -\frac{90}{x^2} + 8\pi x$ (or equivalent) ***A1A1*** **Note**: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given. ***[2 marks]***

(b.ii)

Hence, find the **exact** value of a .

[3]

Markscheme
$\frac{dS}{dx} = 0$ ***(M1)*** $-\frac{90}{a^2} + 8\pi a = 0$ ***A1*** $(a =) \left(\frac{90}{8\pi}\right)^{\frac{1}{3}}$ ***A1*** ***[3 marks]***

(c.i)

Find an expression for $\frac{d^2S}{dx^2}$.

[2]

Markscheme
$\frac{d^2S}{dx^2} = 180x^{-3} + 8\pi$ (or equivalent) ***A1A1*** **Note**: Award ***A1*** for each correct term. Award ***A1A0*** if additional terms are given. ***[2 marks]***

(c.ii)

Use the second derivative of S to justify that S is a minimum when $x = a$.

[2]

Markscheme

EITHER substituting their value of x into their $\frac{d^2S}{dx^2}$ ***(M1)*** $\frac{d^2S}{dx^2} = 75.39822 \ldots = 75.4\ (= 24\pi) > 0$ **OR** sketch of the graph of $\frac{d^2S}{dx^2}$ with their value of x clearly indicated ***(M1)*** $\frac{d^2S}{dx^2} > 0$ at $x = a$ ***A1*** **THEN** therefore S is a minimum ***AG***
[2 marks]

(c.iii)

Find the minimum surface area of the solid.

[2]

Markscheme

attempt to substitute their value of a into S OR use of graph of S ***(M1)*** $88.2401 \ldots$ minimum surface area $= 88.2\ (cm^2)$ ***A1***
[2 marks]

34. 23M.2.SL.TZ1.6

[7]

Markscheme

product of a binomial coefficient, a power of ax^3 and a power of b seen ***(M1)*** evidence of correct term chosen for $n = 8 : r = 2$ (or $r = 6$) OR for $n = 10 : r = 2$ (or $r = 8$) ***(A1)*** correct equations (may include powers of x) ***A1A1*** $a^2b^6 = 448\ (28a^2b^6 = 448 \Rightarrow a^2b^6 = 16)$, $a^2b^8 = 2880\ (45a^2b^8 = 2880 \Rightarrow a^2b^8 = 64)$ attempt to solve their system in a and b algebraically or graphically ***(M1)*** $b = 2$; $a = \frac{1}{2}$ ***A1A1*** **Note:** Award a maximum of ***(M1)(A1)A1A1(M1)A1A0*** for $b = \pm 2$ and/or $a = \pm\frac{1}{2}$. ***[7 marks]***

35. 23M.2.SL.TZ1.7

(a.i)

Find the initial temperature of the tea.

[1]

Markscheme	
$96\ (^\circ)$ (exact)	***A1 [1 mark]***

(a.ii)

Find the temperature of the tea three minutes after it is poured.

[1]

Markscheme	
$79.9970 \ldots 80.0\ (^\circ)$ (accept 80)	***A1 [1 mark]***

(b)

Write down the value of $H'(3)$.

[2]

Markscheme	
$-4.71976\ldots -4.72\ (^\circ C\ \min^{-1})$	***A2 [2 marks]***

(c)

Interpret the meaning of your answer to part (b) in the given context.

[2]

Markscheme	
3 valid descriptors, in any order:	***A2***

- at 3 minutes (or when $t = 3$)
- cooling/decreasing (do not accept "changing")
- $4.72\ ^\circ C\ \text{min}^{-1}$ (must include units) (accept approximately 5 deg/min)

[2 marks]

(d)

After k minutes, the tea will be below $67\ ^\circ C$ and cool enough to drink.

Find the least possible value of k, where $k \in \mathbb{Z}^+$.

[3]

Markscheme

METHOD 1 valid attempt to solve $H(t) = 67$ (accept an inequality) ***(M1)*** eg intersection of graphs, use of logarithms. 6.11058… ***(A1)*** 7 (min) ***A1*** **METHOD 2** valid attempt to find crossover values ***(M1)*** (6, 67.4087…) and (7, 63.8406…) ***(A1)*** 7 (min) ***A1*** ***[3 marks]***

(e)

Find the temperature of the room.

[2]

Markscheme

recognition that $t \to \infty$ ***(M1)*** $21\ (^\circ C)$ ***A1*** ***[2 marks]***

(f)

Find the limit of $H'(t)$ as t approaches infinity.

[2]

Markscheme

METHOD 1 (working with slopes of H) valid attempt to analyse progression of slopes of H ***(M1)*** $\lim_{t\to\infty} H'(t) = 0$ ***A1*** **METHOD 2 (working with H′)** valid attempt to use H' and large values of t. ***(M1)*** $\lim_{t\to\infty} H'(t) = 0$ ***A1 [2 marks]***

36. 23M.2.SL.TZ2.6

(a)

Find the value of t_1.

[2]

Markscheme

recognize that acceleration is zero when $v'(t) = 0$ OR at a local maximum on the graph of v ***(M1)*** $t_1 = 0.394791\ldots$ $t_1 = 0.395$ $\left(= \arctan\left(\frac{5}{12}\right)\right)$ (seconds) ***A1 [2 marks]***

(b)

Find the value of t_2.

[2]

Markscheme

recognition that $v = 0$ ***(M1)*** sketch OR $t = 4.71238\ldots$ OR $t = 10.9955\ldots$ $t_2 = 10.9955\ldots$ $t_2 = 11.0$ $\left(= \frac{7\pi}{2}\right)$ ***A1 [2 marks]***

(c)

Find the distance travelled by the particle between $t = t_1$ and $t = t_2$.

[2]

Markscheme

$\int_{t_1}^{t_2}|v|\,dt$ OR $\int_{0.394791\ldots}^{10.9955\ldots}|v|\,dt$ OR $\int_{0.394791\ldots}^{4.71238\ldots}|v|\,dt$ OR $\int_{4.71238\ldots}^{10.9955\ldots}|v|\,dt$ $(= 6.53806\ldots + 1.29313\ldots)$ OR $\int_{0.394791\ldots}^{4.71238\ldots} v\,dt - \int_{4.71238\ldots}^{10.9955\ldots} v\,dt\ (= 6.53806\ldots - (-1.29313\ldots))$ ***(A1)*** distance = $7.83118\ldots = 7.83$ (m) ***A1 [2 marks]***

37. 23M.2.SL.TZ2.7

(a.i)

Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain.

[4]

Markscheme
swapping x and y, or $h(h^{-1}(x)) = x$ ***(M1)*** $h^{-1}(x) = \frac{x^2+2}{4}$ ***A1*** recognizing range of h is domain of h^{-1} ***(M1)*** Domain: $x \geq 0$ ***A1 [4 marks]***

(a.ii)

Write down the range of $h^{-1}(x)$.

[1]

Markscheme
range of h^{-1} is $y \geq \frac{1}{2}$ ***[1 mark]***

(b)

The graph of h intersects the graph of h^{-1} at two points.

Find the x-coordinates of these two points.

[[N/A]]

Markscheme
$\sqrt{4x-2} = \frac{x^2+2}{4}$ OR $\sqrt{4x-2} = x$ OR $\frac{x^2+2}{4} = x$ ***(M1)*** $x = 0.585786\ldots$, $x = 3..414213\ldots$ $(= 2+\sqrt{2})$ $x = 0.586$, $x = 3.41$ ***A1A1 [3 marks]***

(c)

Find the area enclosed by the graph of h and the graph of h^{-1}.

[2]

Markscheme
attempt to form integral of the difference between $h(x)$ and their h^{-1}, using their limits from part (b) ***(M1)*** $\int_{0..585786\ldots}^{3.414213\ldots}(h(x) - h^{-1}(x))\,dx$ OR $\int_{0..585786\ldots}^{3.414213\ldots}\left(\sqrt{4x-2} - \frac{x^2+2}{4}\right)dx$ OR $6.5996632\ldots - 4.7140452\ldots$ $1.88561\ldots$ $area = 1.89$ ***A1 [2 marks]***

(d)

Find $h'(x)$.

[2]

Markscheme
attempt to use chain rule or power rule ***(M1)*** $h'(x) = 4\cdot\frac{1}{2}(4x-2)^{-\frac{1}{2}}$ $h'(x) = \frac{2}{\sqrt{4x-2}}$ ***A1 [2 marks]***

(e)

Find the value of x for which the graph of h and the graph of h^{-1} have the same gradient.

[3]

Markscheme

EITHER $(h^{-1})'(x) = \frac{x}{2}$ ***(A1)*** equating their $h'(x)$ to the derivative of their $h^{-1}(x)$ and attempting to solve for x ***(M1)*** $\frac{2}{\sqrt{4x-2}} = \frac{x}{2}$ **OR** finding intersection of graphs of their derivatives ***(M2)*** **THEN** $1.772776 \ldots x = 1.77$ ***A1*** ***[3 marks]***

38. 22M.2.SL.TZ2.6

(a)

Determine when the particle changes its direction of motion.

[2]

Markscheme

recognises the need to find the value of t when $v = 0$ ***(M1)***

$t = 1.57079 \ldots \left(= \frac{\pi}{2}\right)$

$t = 1.57 \left(= \frac{\pi}{2}\right)$ (s) ***A1***

[2 marks]

(b)

Find the times when the particle's acceleration is -1.9 m s^{-2}.

[3]

Markscheme

recognises that $a(t) = v'(t)$ ***(M1)***

$t_1 = 2.26277 \ldots, \ t_2 = 2.95736 \ldots$

$t_1 = 2.26, \ t_2 = 2.96$ (s) ***A1A1***

Note: Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \leq t \leq 3$.

[3 marks]

(c)

Find the particle's acceleration when its speed is at its greatest.

[2]

Markscheme
speed is greatest at $t = 3$ ***(A1)***
$a = -1.83778\ldots$
$a = -1.84\ (\text{m s}^{-2})$ ***A1***
[2 marks]

39. 22M.2.SL.TZ2.7

(a)

Find the distance from point A to point C.

[4]

Markscheme
$\text{A}\hat{\text{B}}\text{C} = 27°$ ***(A1)***
attempt to substitute into cosine rule ***(M1)***
$175^2 + 230^2 - 2(175)(230)\cos 27°$ ***(A1)***
$108.62308\ldots$
$\text{AC} = 109\ (\text{m})$ ***A1***
[4 marks]

(b)

Find the area of this piece of land.

[2]

Markscheme
correct substitution into area formula ***(A1)***

$$\frac{1}{2} \times 175 \times 230 \times \sin 27°$$

$$9136.55 \ldots$$

area $= 9140\ (\text{m}^2)$ ***A1***

[2 marks]

(c)

Find CÂB.

[3]

Markscheme

attempt to substitute into sine rule or cosine rule ***(M1)***

$\frac{\sin 27°}{108.623\ldots} = \frac{\sin \hat{A}}{175}$ OR $\cos A = \frac{(108.623\ldots)^2 + 230^2 - 175^2}{2 \times 108.623\ldots \times 230}$ ***(A1)***

$$47.0049 \ldots$$

CÂB $= 47.0°$ ***A1***

[3 marks]

(d)

Find the distance from point B to point D.

[5]

Markscheme

METHOD 1

recognizing that for areas to be equal, AD = DC ***(M1)***

AD $= \frac{1}{2}$AC $= 54.3115 \ldots$ ***A1***

attempt to substitute into cosine rule to find BD ***(M1)***

correct substitution into cosine rule ***(A1)***

$$\text{BD}^2 = 230^2 + 54.3115^2 - 2(230)(54.3115)\cos 47.0049°$$

$$\text{BD} = 197.009 \ldots$$

BD $= 197\ (\text{m})$ ***A1***

METHOD 2

correct expressions for areas of triangle BDA and triangle BCD using BD ***A1***

$\frac{1}{2} \times \text{BD} \times 230 \times \sin x°$ and $\frac{1}{2} \times \text{BD} \times 175 \times \sin(27 - x)°$ OR
$\frac{1}{2} \times \text{BD} \times 230 \times \sin(27 - x)°$ and $\frac{1}{2} \times \text{BD} \times 175 \times \sin x°$
correct equation in terms of x ***(A1)***
$175 \sin(27 - x) = 230 \sin x$ or $175 \sin x = 230 \sin(27 - x)$
$x = 11.6326 \ldots$ or $x = 15.3673 \ldots$ ***(A1)***
substituting their value of x into equation to solve for BD ***(M1)***
$\frac{1}{2} \times \text{BD} \times 230 \times \sin 11.6326 \ldots = \frac{1}{2} \times \text{BD} \times 175 \times \sin 15.3673 \ldots$ or
$\frac{1}{2} \times \text{BD} \times 230 \times \sin 11.6326 \ldots = \frac{1}{2} \times 9136.55 \ldots$
$\text{BD} = 197$ (m) ***A1***

[5 marks]

40. 21M.2.SL.TZ1.7

(a.i)

Find the value of Amelia's investment after 5 years to the nearest hundred dollars.

[3]

Markscheme

EITHER $9000 \times \left(1 + \frac{7}{100}\right)^5$ ***(A1)*** $12622.965 \ldots$ ***(A1)*** **OR** $n = 5$
$\text{I}\% = 7$
$\text{PV} = \mp 9000$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(A1)***
$\pm 12622.965 \ldots$ ***(A1)*** **THEN** $(\$)\ 12600$ ***A1*** ***[3 marks]***

(a.ii)

Determine the number of years required for Amelia's investment to reach the target.

[2]

Markscheme

EITHER $9000\left(1 + \frac{7}{100}\right)^x = 20000$ ***(A1)*** **OR** $\text{I}\% = 7$
$\text{PV} = \mp 9000$

FV $= \pm 20000$
P/Y $= 1$
C/Y $= 1$ ***(A1)*** **THEN** $= 12$ (years) ***A1*** ***[2 marks]***

(b)

Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years.

[3]

Markscheme

METHOD 1 attempt to substitute into compound interest formula (condone absence of compounding periods) ***(M1)*** $9000\left(1+\frac{r}{100\times 12}\right)^{12\times 10} = 20000$ $8.01170\ldots$ ***(A1)*** $r = 8.02\,(\%)$ ***A1*** **METHOD 2** $n = 10$
PV $= \pm 9000$
FV $= \mp 20000$
P/Y $= 1$
C/Y $= 12$
$r = 8.01170\ldots$ ***(M1)(A1)*** **Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for $(r =)\ 8.01170\ldots$ $r = 8.02\,(\%)$ ***A1*** ***[3 marks]***

(c.i)

Show that Chris will never reach the target if his initial deposit is $\$9000$.

[5]

Markscheme

recognising geometric series (seen anywhere) ***(M1)*** $r = \frac{4500}{9000}\left(=\frac{1}{2}\right)$ ***(A1)*** **EITHER** considering S_∞ ***(M1)***
$\frac{9000}{1-0.5}\,(=18000)$ ***A1*** correct reasoning that $18000 < 20000$ ***R1*** **Note:** Accept $S_\infty < 20000$ only if S_∞ has been calculated. **OR** considering S_n for a large value of n, $n \geq$

80 ***(M1)*** **Note:** Award ***M1*** only if the candidate gives a valid reason for choosing a value of n, where $50 \leq n < 80$. correct value of S_n for their n ***A1*** valid reason why Chris will not reach the target, which involves their choice of n, their value of S_n and Chris' age OR using two large values of n to recognize asymptotic behaviour of S_n as $n \to \infty$. ***R1*** **Note:** Do not award the ***R*** mark without the preceding ***A*** mark. **THEN** Therefore, Chris will never reach the target. ***AG*** ***[5 marks]***

(c.ii)

Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar.

[3]

Markscheme
recognising geometric sum ***M1*** $\frac{u_1(1-0.5^5)}{0.5} = 20000$ ***(A1)*** 10322.58 ... ($) 10323 ***A1*** ***[3 marks]***