SL AA PAPER 1 (Questions 6 – 7) no GDC

1. 25M.1.SL.TZ1.6

Consider a sequence of ten rectangular picture frames $F_1, F_2, \dots, F_9, F_{10}$.

Picture frame F_1 has width $4\ cm$ and height $5\ cm$.

The width and height of picture frame F_n, are each increased by $50\ \%$ to generate the width and height of the next picture frame F_{n+1}, for $n \in \mathbb{Z}^+, 1 \leq n \leq 9$.

(a.i) Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1}\ cm^2$. [2]

(a.ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^{a}-1\right)\ cm^2$, where $p \in \mathbb{Q}^+, a \in \mathbb{Z}^+$. [3]

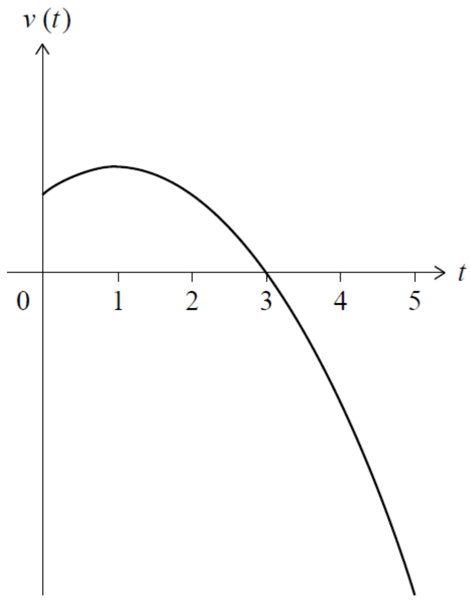
(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^{4}\ cm^2$, where $q \in \mathbb{Q}^+$. [3]

2. 25M.1.SL.TZ1.7

An object moves in a straight line.

Its velocity vm s^{-1}, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \leq t \leq 5$.

The graph of v is shown in the following diagram.



The graph of v has a local maximum point where $t = 1$ and intersects the t-axis at $t = 3$.

(a) Determine the object's

(a.i) maximum velocity; [2]

(a.ii) maximum speed. [2]

At $t = T$, the object changes direction.

(b.i) Write down the value of T. [1]

(b.ii) Find the distance travelled by the object in the first T seconds. [4]

(c) Determine whether the object returns to its initial position during the time period $0 \leq t \leq 5$, justifying your answer. [4]

3. 24N.1.SL.TZ1.6

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

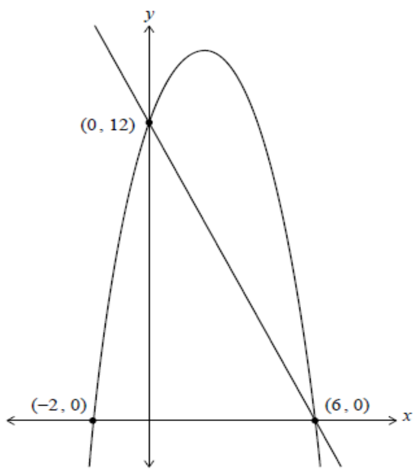
Find the value of k such that $u_k = 0$. [6]

4. 24N.1.SL.TZ1.7

The following diagram shows parts of the graphs of two functions f and g.

The graph of f is linear, has an x-intercept at $(6,\ 0)$ and a y-intercept at $(0,\ 12)$.

The graph of g has x-intercepts at $(-2,\ 0)$ and $(6,\ 0)$ and a y-intercept at $(0,\ 12)$.



(a) Write down the equation for f in the form $f(x) = mx + c$. [2]

The function g is given by $g(x) = -x^2 + bx + 12$, where b is a real constant.

(b) Find the value of b. [3]

(c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^6 (-x^2 + 6x)\, dx$. [2]

(d) Hence, find the area of the region enclosed by the graph of f and the graph of g. [4]

Point P is on the graph of g. The tangent to the graph of g at P is parallel to the graph of f.

(e) Find the coordinates of P. [5]

5. 25M.1.SL.TZ2.6

Consider the function $f(x) = \sqrt{x^2 \ln x + 4 - x^2}$, where $x \in \mathbb{R},\ x > 0$.

(a) Show that the distance, l, between the origin and any point on the graph of f is given by $l = \sqrt{x^2 \ln x + 4}$. [1]

(b) Hence, find the x-coordinate of the point on the graph of f which is closest to the origin.

[6]

6. 25M.1.SL.TZ2.7

A discrete random variable, X, has the following probability distribution, where $a > 0$ and k is a constant.

x	0	a	$2a$	$3a$
$\mathrm{P}(X=x)$	k	$3k^2$	$2k^2$	k^2

(a) Show that $k = \frac{1}{3}$. [5]

(b) Find $P\ (X < 3a)$. [2]

(c) Find $P\ (X \geq a \ \ X < 3a)$. [3]

(d) Given that $E\ (X) = 20$, find the value of a. [3]

7. 24M.1.SL.TZ1.6

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

(a) Find an expression for S_n in the form $\frac{a^n-1}{b}$, where $a,\ b \in \mathbb{Z}^+$. [1]

(b) Hence, show that $S_1 + S_2 + S_3 + \cdots + S_n = \frac{10(10^n-1)-9n}{81}$. [4]

8. 24M.1.SL.TZ1.7

Consider the curve with equation $y = x^3 - x^2 - x + 1$.

(a) Find

(a.i) $\frac{dy}{dx}$; [2]

(a.ii) $\frac{d^2y}{dx^2}$. [1]

The curve has a local maximum at A.

(b) Find the coordinates of A, using your answer to part (a)(ii) to justify your answer. [6]

The curve has a point of inflexion at B.

(c) Find the x-coordinate of B. [2]

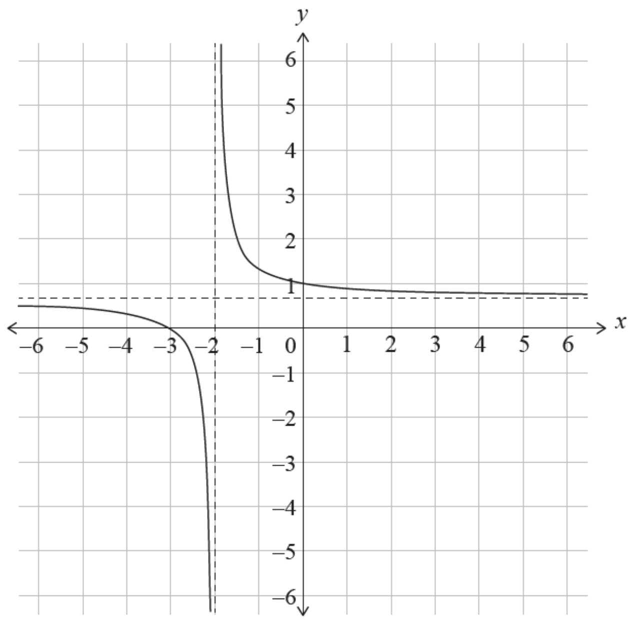
The line L is the normal to the curve at the point $(0,\ 1)$.

(d) Find the equation of L. [3]

9. 24M.1.SL.TZ2.6

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R},\ x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R},\ m \neq 0$.

(b.i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$. [1]

(b.ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x. [4]

(b.iii)Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$.

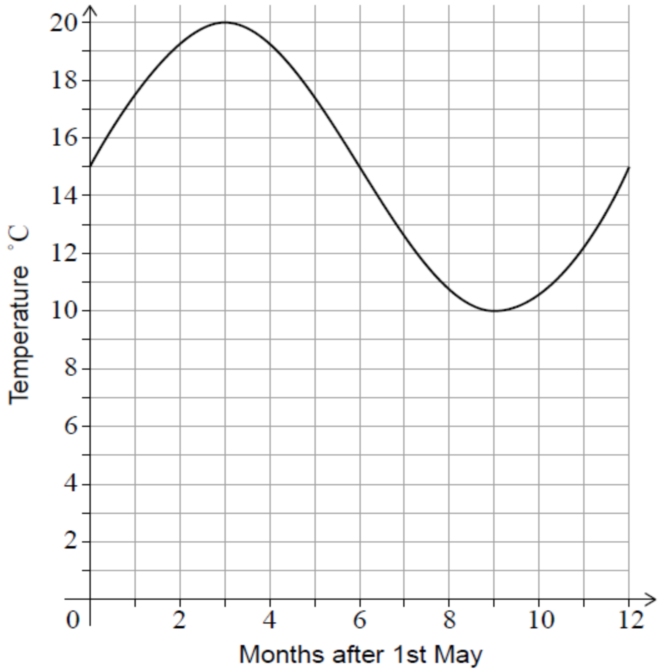
[2]

10. 24M.1.SL.TZ2.7

Alex only swims in the sea if the water temperature is at least $15°C$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a \sin bx + c$ for $0 \leq x \leq 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Write down the value of

(b.i) a; [1]

(b.ii) c. [1]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5 \sin \frac{\pi}{6} x + 11$, where $0 \leq x \leq 12$ and x is the number of months after 1st May.

(b) Using this new model $g(x)$

(c.i) find the midday water temperature on 1st October, five months after 1st May. [3]

(c.ii) show that the midday water temperature is never warm enough for Alex to swim. [3]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]

11. 23N.1.SL.TZ1.6

The binomial expansion of $(1+kx)^n$ is given by $1+\frac{9x}{2}+15k^2x^2+\cdots+k^nx^n$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of *n* and the value of *k*. [6]

12. 23N.1.SL.TZ1.7

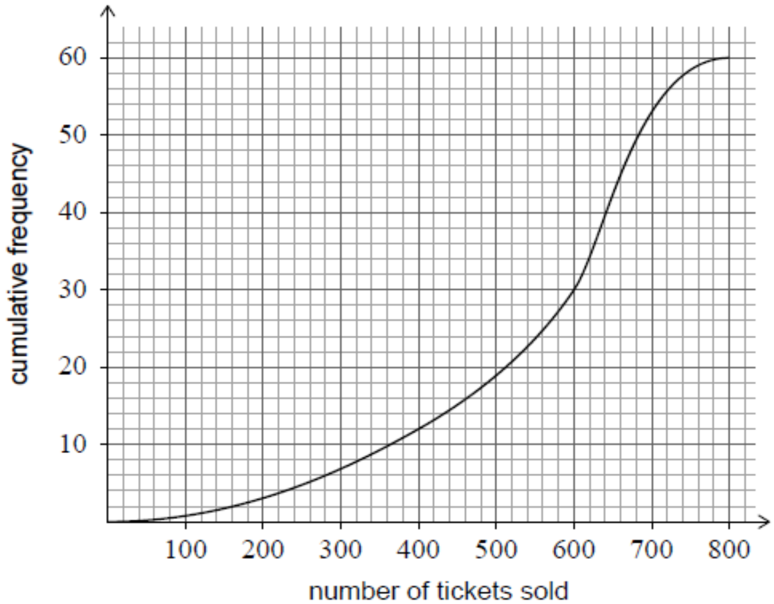
A ballet company performs *The Sleeping Beauty* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n, at each performance is shown in the following frequency table.

Number of tickets sold, n	**Number of performances**
$0 < n \leq 200$	3
$200 < n \leq 400$	p
$400 < n \leq 600$	18
$600 < n \leq 800$	30

(a) (a.i) Find the value of p. [1]

(a.ii) Write down the modal class. [1]

The following cumulative frequency diagram also displays these data.



(b) Use the cumulative frequency curve to estimate

(b.i) the median number of tickets sold. [1]

(b.ii) the number of performances where at least 80 % of the tickets were sold. [3]

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

(c) (c.i)

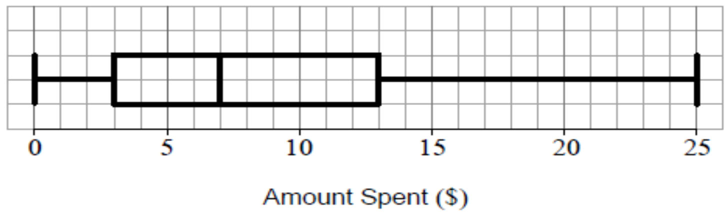
State one disadvantage of the company surveying only the first 5 % of the audience as they leave the theatre. [1]

(c.ii) Describe briefly how the company could collect feedback from 5 % of the audience using the systematic sampling method. [2]

(c.iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience. [1]

Last year 36 000 tickets were sold to *The Sleeping Beauty*.

(d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.



(d.i) Estimate the number of people who spent between $3 and $25. [2]

(d.ii) Half the audience spent less than $\$a$. Estimate the value of a. [1]

This year the company will again give 60 performances and expects to sell 18 additional tickets for each performance.

(e) (e.i) Calculate the mean number of tickets the company expects to sell this year for each performance. [3]

(e.ii) State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance. [1]

13. 23M.1.SL.TZ2.6

The functions f and g are defined for $x \in \mathbb{R}$ by

$f(x) = ax + b$, where $a, b \in \mathbb{Z}$

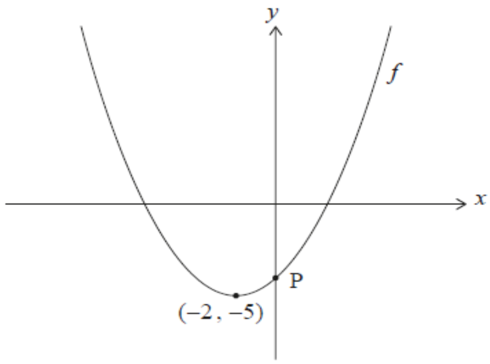
$g(x) = x^2 + x + 3$.

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$. [7]

14. 23M.1.SL.TZ2.7

The following diagram shows part of the graph of a quadratic function f.

The vertex of the parabola is $(-2, -5)$ and the y-intercept is at point P.



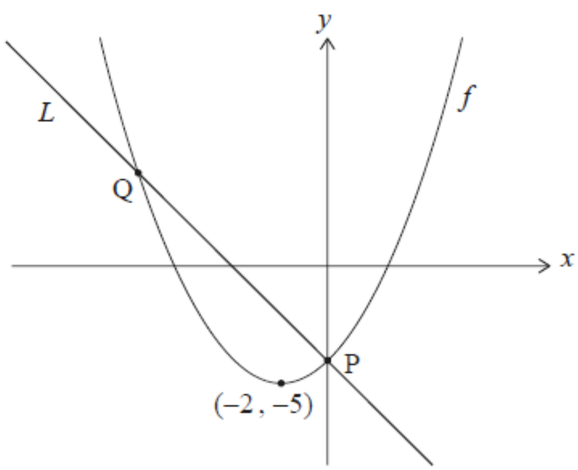
(a) Write down the equation of the axis of symmetry. [1]

The function can be written in the form $f(x) = \frac{1}{4}(x-h)^2 + k$, where $h, k \in \mathbb{Z}$.

(b) Write down the values of h and k. [2]

(c) Find the y-coordinate of P. [2]

In the following diagram, the line L is normal to the graph of f at point P.



(d) Find the equation of the line L, in the form $y = ax + b$. [3]

The line L intersects the graph of f at a second point, Q, as shown above.

(e) Calculate the distance between P and Q. [8]

15. 22M.1.SL.TZ1.6

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

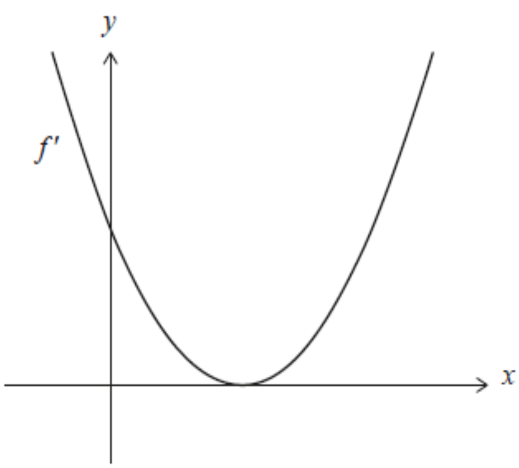
(a) Describe these two transformations. [2]

(b) The y-intercept of the graph of g is at $(0,\ r)$.

Given that $g(x) \geq 7$, find the smallest value of r. [5]

16. 22M.1.SL.TZ1.7

A function, f, has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f'.



The graph of f' has an axis of symmetry $x = q$.

(a) Find the value of q. [2]

The vertex of the graph of f' lies on the x-axis.

(b.i) Write down the value of the discriminant of f'. [1]

(b.ii) Hence or otherwise, find the value of p. [3]

(c) Find the value of the gradient of the graph of f' at $x = 0$. [3]

(d) Sketch the graph of f'', the second derivative of f. Indicate clearly the x-intercept and the y-intercept. [2]

The graph of f has a point of inflexion at $x = a$.

(e.i) Write down the value of a. [1]

(e.ii) Find the values of x for which the graph of f is concave-down. Justify your answer. [2]

17. 22M.1.SL.TZ2.6

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \cdots + 1$ where $x \neq 0$ and $a,\ b \in \mathbb{Z}^+$.

(a) Show that $b = 21$. [2]

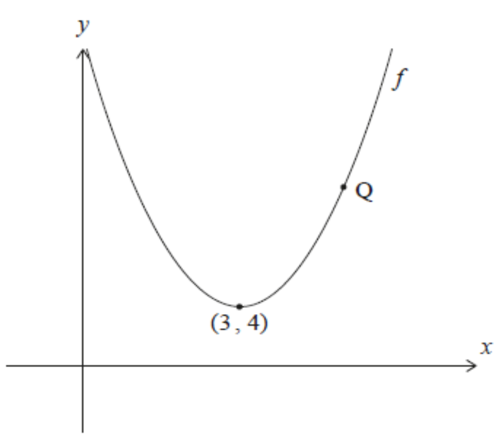
(b) The third term in the expansion is the mean of the second term and the fourth term in the expansion.

Find the possible values of x. [5]

18. 22M.1.SL.TZ2.7

The following diagram shows part of the graph of a quadratic function f.

The graph of f has its vertex at $(3,\ 4)$, and it passes through point Q as shown.



(a) Write down the equation of the axis of symmetry. [1]

The function can be written in the form $f(x) = a(x-h)^2 + k$.

(b.i) Write down the values of h and k. [2]

(b.ii) Point Q has coordinates $(5,\ 12)$. Find the value of a. [2]

The line L is tangent to the graph of f at Q.

(c) Find the equation of L. [4]

Now consider another function $y = g(x)$. The derivative of g is given by $g'(x) = f(x) - d$, where $d \in \mathbb{R}$.

(d) Find the values of d for which g is an increasing function. [3]

(e) Find the values of x for which the graph of g is concave-up. [3]

19. 22N.1.SL.TZ0.6

Events A and B are such that $\text{P}(A) = 0.3$ and $\text{P}(B) = 0.8$.

(a) Determine the value of $\text{P}(A \cap B)$ in the case where the events A and B are independent.

[1]

(b) Determine the minimum possible value of $\text{P}(A \cap B)$. [3]

(c) Determine the maximum possible value of $\text{P}(A \cap B)$, justifying your answer. [2]

20. 22N.1.SL.TZ0.7

(a) The graph of a quadratic function f has its vertex at the point $(3,\ 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f(x) = a(x-h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t-1)x - p$ where $x \in \mathbb{R}$ and $p,\ t \in \mathbb{R},\ p \neq 0$.

In the case where $g(-3) = g(1) = 4$,

(b.i) find the value of p and the value of t. [4]

(b.ii) find the range of g. [3]

(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R},\ p \neq 0$.

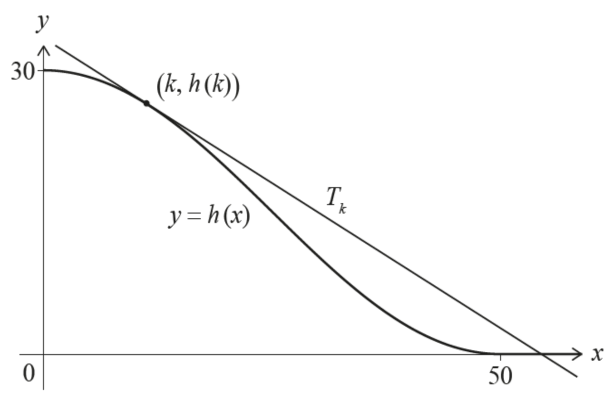
Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t-1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

PAPER 2 With GDC

21. 25M.2.SL.TZ1.6

Consider the function $h(x) = 15\cos\left(\frac{\pi x}{50}\right) + 15$, where $0 \leq x \leq 50$.

The tangent, T_k, to the curve $y = h(x)$ at the point $\left(k,\ h(k)\right)$ is shown on the following diagram.



(a) Find the gradient of T_k in terms of k. [3]

Consider the case where the angle between T_k and the x-axis is $\frac{\pi}{8}$ radians.

(b) Find the possible values of k. [3]

22. 25M.2.SL.TZ1.7

Lynn is playing a game with two unbiased six-sided dice, each with faces marked with the integers from 1 to 6.

In each round, she throws both dice once. The outcomes can be displayed in the following sample space diagram, which has been partially completed:

Die 2

Die 1	1	2	3	4	5	6
1						
2		2,2				
3				3,4		
4						
5			5,3			
6						

Lynn scores points according to the following rules.

- If the two dice show the same score, she scores 10 points.
- If the two dice show scores which have a difference of one, for example the scores 4 and 5 in any order, she scores 5 points.
- Otherwise, she scores 0 points.

(a) Show that the probability that Lynn scores 5 points in one round is $\frac{5}{18}$. [2]

(b) Find the probability that Lynn scores no points in one round. [2]

The random variable X represents the number of points Lynn scores in one round.

(c) Find E (X). [4]

(d) Hence, estimate the total number of points that Lynn scores if she plays 90 rounds. [2]

A prize is awarded to any player who scores more than 40 points in total.

Lynn plays exactly five rounds.

(e) Find the probability that Lynn wins a prize. [4]

23. 24N.2.SL.TZ1.6

Consider the function $h(x) = \log_{10}(3x^2 - rx + r - 2)$, where $x \in \mathbb{R}$.

Find the possible values of r. [5]

24. 24N.2.SL.TZ1.7

MyLife is a social media platform with 89.8 million users, all aged 12 years old and above. The following frequency table shows the number of users by age group.

Age, a (years)	**Millions of users**
$12 \le a < 18$	5.8
$18 \le a < 35$	42.7
$35 \le a < 55$	26.3
$55 \le a < 75$	12.9
$75 \le a < 95$	2.1
Total	**89.8**

(a) Find the percentage of *MyLife* users aged 55 years or older. [1]

(b) A sample of 1000 *MyLife* users is chosen at random. Find the probability that fewer than 150 of them are aged 55 years or older. [3]

(c) Given that a *MyLife* user chosen at random is 55 years or older, find the probability that they are 75 years or older. [4]

(d) List the mid-interval value for each class interval. [1]

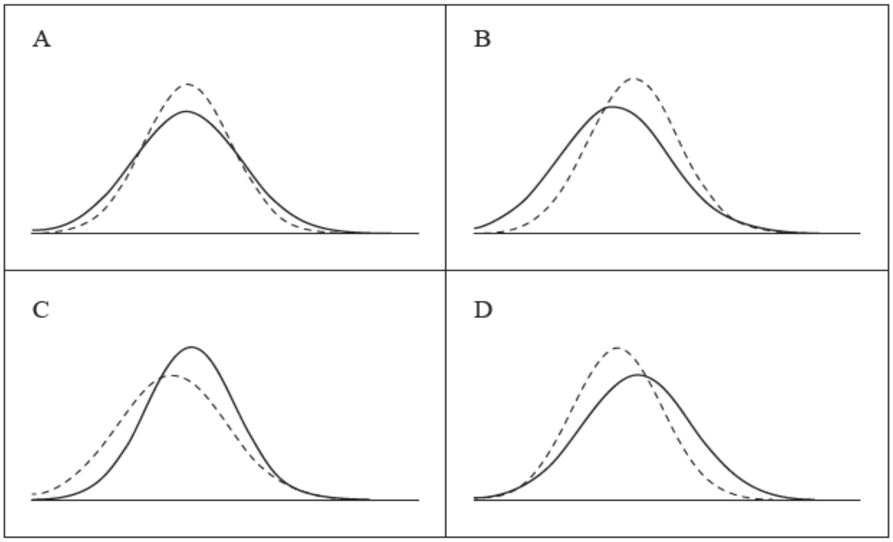
(e) Hence, for *MyLife* users, estimate

(e.i) the mean age; [1]

(e.ii) the variance of the ages. [2]

A different social media platform, *SmallTalk*, reports that its users have a mean age of 29.9 years and a variance of 137 years^2.

The following four diagrams represent age distributions.



(f.i) Identify the diagram which best represents the age distributions for the users of *MyLife* and *SmallTalk*. [1]

(f.ii) In your chosen diagram, identify which social media platform is represented by the dotted line. [1]

25. 25M.2.SL.TZ3.6

Consider the function $f(x) = \frac{(2x+a)^3}{(x+5)^2}$, where $x \neq -5$ and $a \in \mathbb{R}^+$.

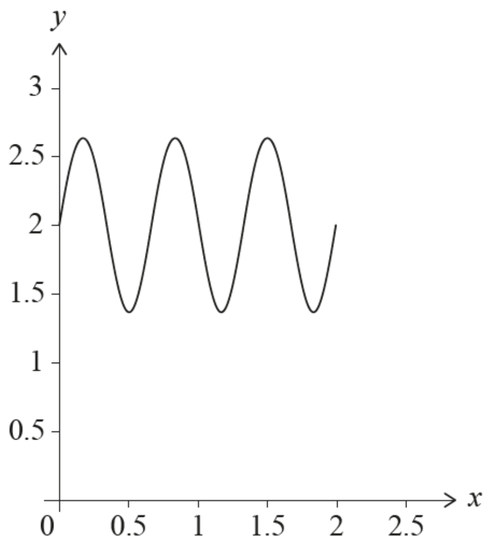
(a) Find an expression for $f'(x)$, in terms of a. [3]

When $x = 1$, the tangent to the graph of f makes an angle of $70°$ to the horizontal.

(b) Find the two possible values of a. [4]

26. 25M.2.SL.TZ3.7

Consider the function $f(x) = \frac{2}{\pi}\sin(3\pi x) + 2$, where $0 \leq x \leq 2$. The following diagram shows the graph of f.



(a.i) Write down the amplitude of f. [1]

(a.ii) Find the period of f. [2]

(b) The point P has coordinates $(1.63,\ 2.16)$. State whether P lies above, below or on the graph of f. Justify your answer. [3]

The line L_1 has equation $x - 6y + 11 = 0$.

(c) Write down the gradient of the line L_1. [1]

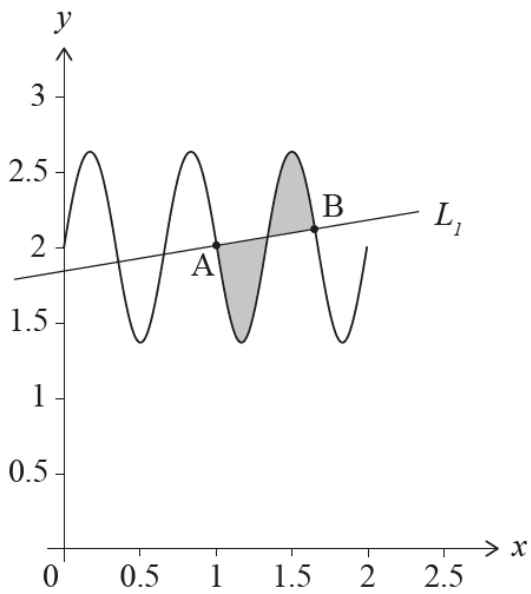
The line L_1 is normal to the graph of f at point $A\ (1,\ 2)$.

The line L_2 is tangent to the graph of f at A.

(d.i) Find the gradient of L_2. [2]

(d.ii) Hence, or otherwise, find the equation of L_2. [1]

The line L_1 intersects the graph of f at another point B, where the x-coordinate of B is greater than 1.5. This is shown in the following diagram.



(e) Find the coordinates of B. [2]

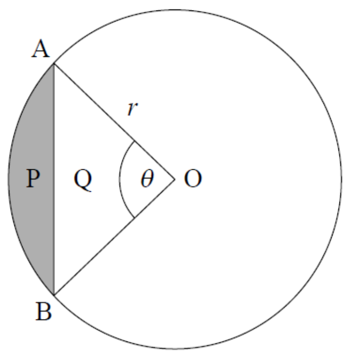
The shaded region is enclosed by the graph of f and the line L_1 between A and B.

(f) Find the area of the shaded region. [3]

27. 25M.2.SL.TZ2.6

The following diagram shows a circle with centre O and radius $r\ cm$. Points A and B lie on the circle and $A\hat{O}B = \theta$ radians.

Sector OAB is divided into two regions, a shaded segment P and a triangle Q.



The area of the shaded segment P is $12.8\ cm^2$.

The areas of P and Q are in the ratio $3:5$.

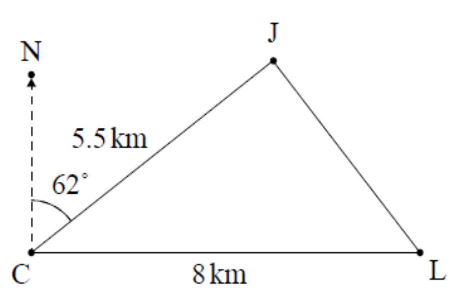
Find the value of r. [6]

28. 25M.2.SL.TZ2.7

A lighthouse, L, is located 8 kilometres due East of a coastguard station, C, on a straight stretch of coastline.

The coastguard station sees a Jet Ski, J, on a bearing of $062°$ and at a distance of 5.5 kilometres. This is shown on the following diagram.

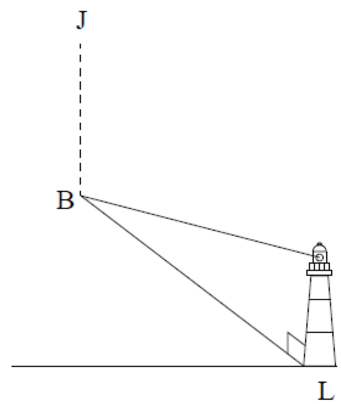
diagram not to scale



(a) Find JL. [4]

While travelling due South, the Jet Ski breaks down at point B, before it reaches the coastline. The position of the Jet Ski at B and the lighthouse are shown in the following diagram.

diagram not to scale



From the top of the 60-metre-tall lighthouse, the angle of depression to the Jet Ski at B, is measured to be $0.94°$.

(b) Find BL. [3]

The bearing from the Jet Ski at B to the lighthouse is $121°$.

(c) Find the bearing from L to B. [2]

The jet-skier sets off a distress flare which is seen at the lighthouse and the coastguard station at the same time.

The lighthouse has a small rescue boat which travels at a speed of $48\ kmh^{-1}$.

(d) Find the time, in minutes, for the lighthouse rescue boat to reach the Jet Ski at B. [2]

The coastguard rescue boat travels at a speed of $55\ kmh^{-1}$ and sets out at the same time as the lighthouse rescue boat.

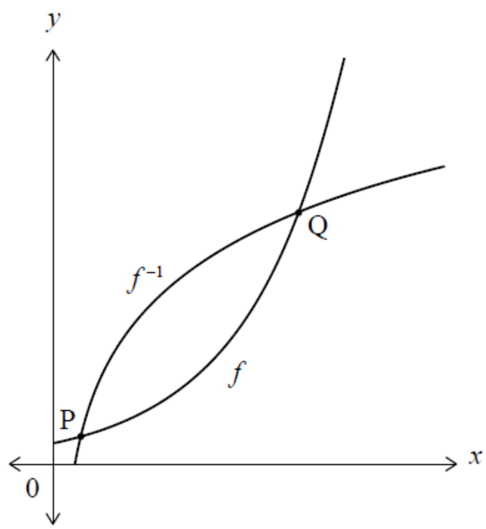
(e) Determine which rescue boat reaches the Jet Ski first. Justify your answer. [4]

29. 24M.2.SL.TZ1.7

Consider the function defined by $f(x) = \frac{3}{2}e^{x-2},\ 0 \leq x \leq 4.$

(a) Show that the inverse function is given by $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$. [3]

The graphs of f and f^{-1} intersect at two points P and Q, as shown on the following diagram.



(b) Find PQ. [3]

The graph of f is reflected in the x-axis and then translated parallel to the y-axis by 5 units in the positive direction to give the graph of a function g.

(c) Write down

(c.i) an expression for $g(x)$; [2]

(c.ii) the domain of g. [1]

(d) Solve the equation $f(x) = g(x)$. Give your answer in the form $x = a + \ln b$, where $a,\ b\ \in \mathbb{Q}$. [3]

30. 24M.2.SL.TZ2.6

Consider a random variable X such that $X \sim B(n,\ 0.25)$.

Determine the least value of n such that $P(X \geq 1) > 0.99$. [5]

31. 24M.2.SL.TZ2.7

A lake contains a type of fish called carp. The lengths, $L\ cm$, of the carp can be modelled by a normal distribution with mean $45.6\ cm$ and standard deviation $4.2\ cm$.

According to this model, carp with a length between $41.4\ cm$ and $k\ cm$ lie within one standard deviation of the mean.

(a) Write down the value of k. [2]

(b) Find the probability that a randomly selected carp is greater than $48\ cm$ in length. [2]

(c) It is known that $99\ \%$ of carp in the lake have a length greater than $x\ cm$. Find the value of x. [2]

(d) Consider a random sample of 100 carp from the lake.

(d.i) Find the expected number of carp with lengths between $40\ cm$ and $56\ cm$. [3]

(d.ii) Find the probability that in this sample, exactly 95 carp have a length between $40\ cm$ and $56\ cm$. [2]

A large sample of carp from the lake is studied. The length of each fish is measured and recorded correct to the nearest $0.1\ cm$.

(e) Find the probability that a randomly selected carp has a length recorded as $45.6\ cm$. [3]

32. 23N.2.SL.TZ2.6

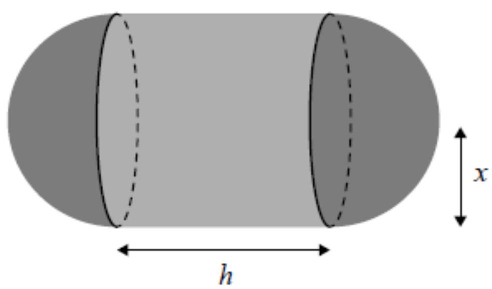
The following table shows the probability distribution of a discrete random variable X, where $a,\ k \in \mathbb{R}^+$.

x	1	2	3	4
$P(X=x)$	k	k^2	a	k^3

Given that $E(X) = 2.3$, find the value of a. [5]

33. 23N.2.SL.TZ2.7

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height $h\ cm$ and radius $x\ cm$. The hemispheres fit exactly onto either end of the cylinder.



The volume of the cylinder is $45\ cm^3$.

(a) Show that the total surface area, $S\ cm^2$, of the solid is given by $S = \frac{90}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (b.i) Find an expression for $\frac{dS}{dx}$. [2]

(b.ii) Hence, find the **exact** value of a . [3]

(c.i) Find an expression for $\frac{d^2S}{dx^2}$. [2]

(c.ii) Use the second derivative of S to justify that S is a minimum when $x = a$. [2]

(c.iii) Find the minimum surface area of the solid. [2]

34. 23M.2.SL.TZ1.6

The coefficient of x^6 in the expansion of $(ax^3 + b)^8$ is 448.

The coefficient of x^6 in the expansion of $(ax^3 + b)^{10}$ is 2880.

Find the value of a and the value of b, where $a\,,\, b > 0$. [7]

35. 23M.2.SL.TZ1.7

The temperature of a cup of tea, t minutes after it is poured, can be modelled by $H(t) = 21 + 75e^{-0.08t},\ t \geq 0$. The temperature is measured in degrees Celsius ($^\circ C$).

(a.i) Find the initial temperature of the tea. [1]

(a.ii) Find the temperature of the tea three minutes after it is poured. [1]

(b) Write down the value of $H'(3)$. [2]

(c) Interpret the meaning of your answer to part (b) in the given context. [2]

(d) After k minutes, the tea will be below 67 $^\circ C$ and cool enough to drink.

Find the least possible value of k, where $k \in \mathbb{Z}^+$. [3]

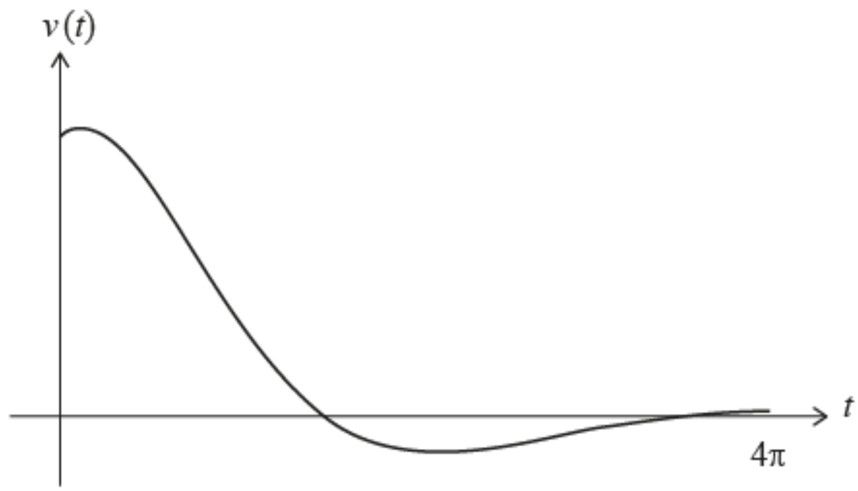
As the tea cools, $H(t)$ approaches the temperature of the room, which is constant.

(e) Find the temperature of the room. [2]

(f) Find the limit of $H'(t)$ as t approaches infinity. [2]

36. 23M.2.SL.TZ2.6

A particle moves in a straight line such that its velocity, $v\,m\,s^{-1}$, at time t seconds is given by $v(t) = 4e^{-\frac{t}{3}}\cos\left(\frac{t}{2} - \frac{\pi}{4}\right)$, for $0 \le t \le 4\pi$. The graph of v is shown in the following diagram.



Let t_1 be the first time when the particle's **acceleration** is zero.

(a) Find the value of t_1. [2]

Let t_2 be the **second** time when the particle is instantaneously at rest.

(b) Find the value of t_2. [2]

(c) Find the distance travelled by the particle between $t = t_1$ and $t = t_2$. [2]

37. 23M.2.SL.TZ2.7

Consider the function $h(x) = \sqrt{4x - 2}$, for $x \geq \frac{1}{2}$.

(a.i) Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain. [4]

(a.ii) Write down the range of $h^{-1}(x)$. [1]

(b) The graph of h intersects the graph of h^{-1} at two points.

Find the x-coordinates of these two points.

(c) Find the area enclosed by the graph of h and the graph of h^{-1}. [2]

(d) Find $h'(x)$. [2]

(e) Find the value of x for which the graph of h and the graph of h^{-1} have the same gradient.

[3]

38. 22M.2.SL.TZ2.6

A particle moves in a straight line such that its velocity, v m s^{-1}, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4}$, $0 \leq t \leq 3$.

(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is -1.9 m s^{-2}. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

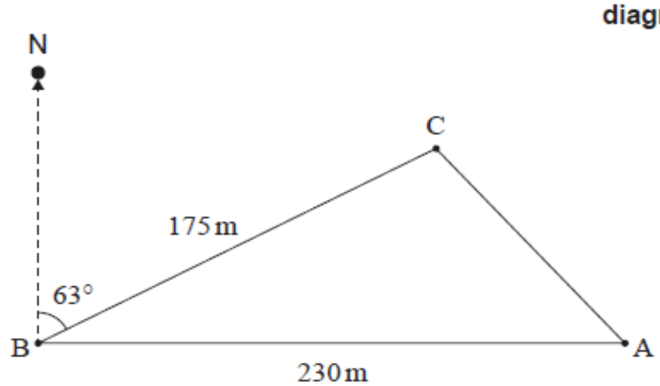
39. 22M.2.SL.TZ2.7

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of 063° to reach point C.

This is shown in the following diagram.

diagram not to scale



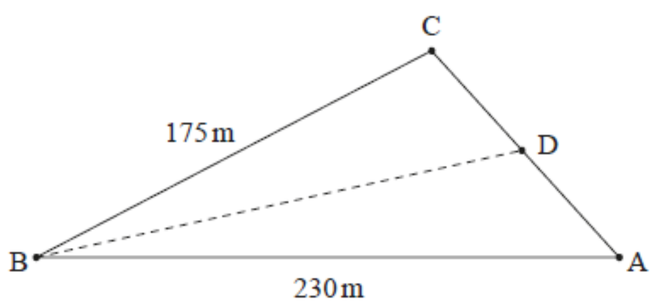
(a) Find the distance from point A to point C. [4]

(b) Find the area of this piece of land. [2]

(c) Find CÂB. [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale



(d) Find the distance from point B to point D. [5]

40. 21M.2.SL.TZ1.7

Two friends Amelia and Bill, each set themselves a target of saving $\$20\,000$. They each have $\$9000$ to invest.

Amelia invests her $\$9000$ in an account that offers an interest rate of 7% per annum compounded **annually**.

(a.i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars. [3]

(a.ii) Determine the number of years required for Amelia's investment to reach the target.

[2]

(b) Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

A third friend Chris also wants to reach the $\$20\,000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

(c.i) Show that Chris will never reach the target if his initial deposit is $\$9000$. [5]

(c.ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar.

[3]