

PAPER 1 QUESTIONS (SHORT QUESTIONS)

Q1. Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = Ae^{kt}$$

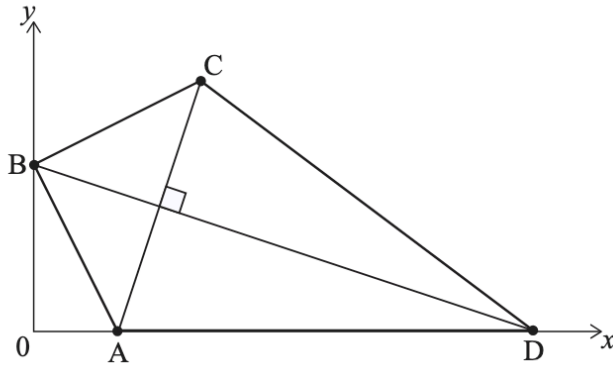
where P is the area covered by mould in mm^2 , t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is 112 mm^2 at the start of the experiment and 360 mm^2 after 5 days.

- (a) Write down the value of A .
- (b) Find the value of k .

Q2. Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm. The coordinates of A, B and C are (2, 0), (0, 4) and (4, 6) respectively. Point D lies on the x -axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



- (a) Find the gradient of the line through A and C.
- (b) Write down the gradient of the line through B and D.
- (c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers.
- (d) Write down the x -coordinate of point D.

Q3. Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter x cm.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \leq x \leq 14$$

(a) Find the range of h .

The function h^{-1} is the inverse function of h .

(b) (i) Find $h^{-1}(10)$.

(ii) In the context of the question, interpret your answer to part (b)(i).

(iii) Write down the range of h^{-1} .



Q4. Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t , the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t -axis.

The rate, R , is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

(a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours.

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

(b) Find the percentage error of the estimate found in part (a).

Q5. The pH of a solution measures its acidity and can be determined using the formula $pH = -\log_{10} C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

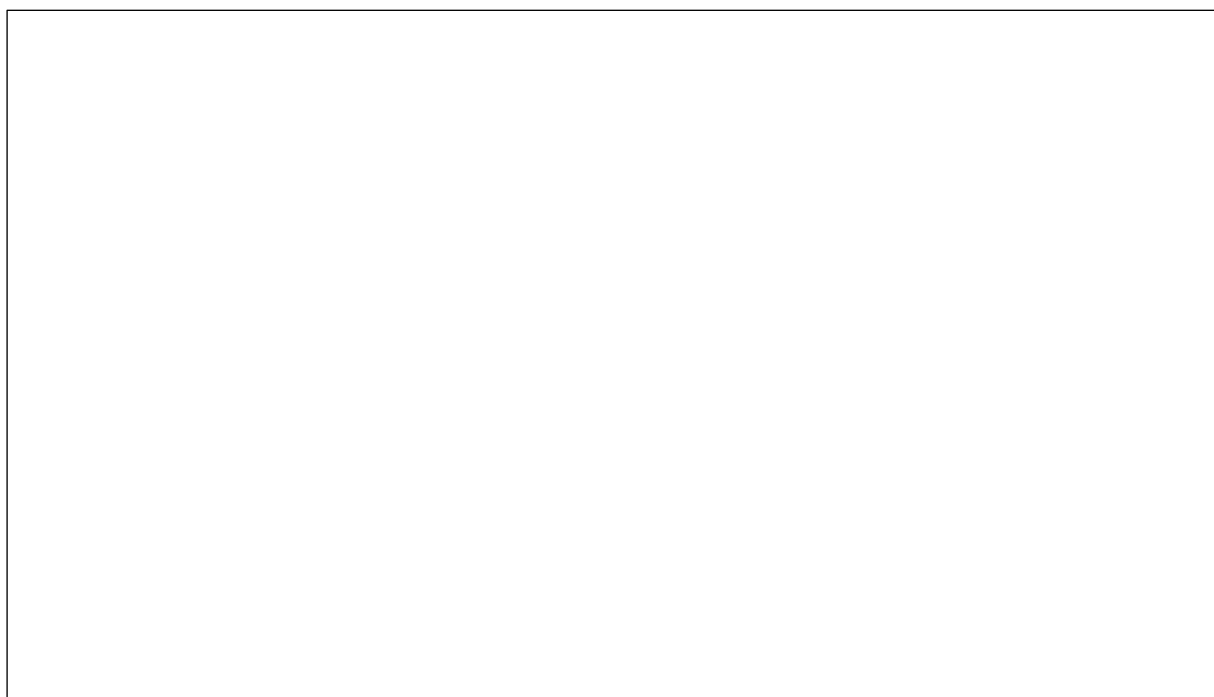
The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

(a) Calculate the pH of the coffee.

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

(b) Determine whether the unknown liquid is more or less acidic than the coffee.

Justify your answer mathematically.



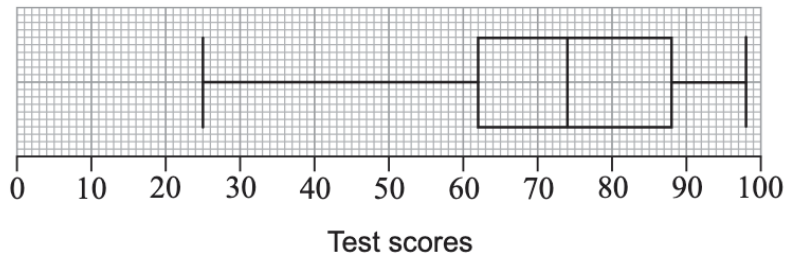
Q6. A college runs a mathematics course in the morning. Scores for a test from this class are shown below.

25 33 51 62 63 63 70 74 79 79 81 88 90 90 98

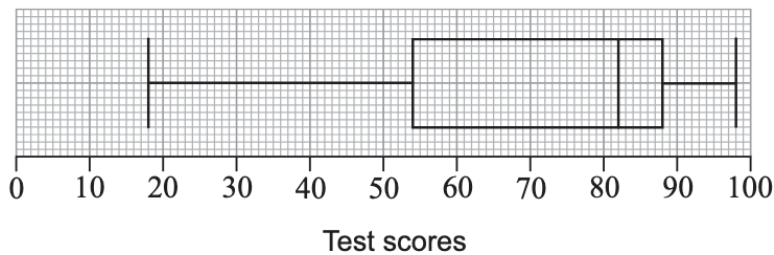
For these data, the lower quartile is 62 and the upper quartile is 88.

a) Show that the test score of 25 would not be considered an outlier.

The box and whisker diagram showing these scores is given below.



Another mathematics class is run by the college during the evening. A box and whisker diagram showing the scores from this class for the same test is given below.



A researcher reviews the box and whisker diagrams and believes that the evening class performed better than the morning class.

(b) With reference to the box and whisker diagrams, state one aspect that may support the researcher's opinion and one aspect that may counter it.

Q7. The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, \$)
Adult	15
Child	10
Student	12

A total of 600 tickets were sold.

The total amount of money from ticket sales was \$7816.

There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x , the number of child tickets sold be y , and the number of student tickets sold be z .

(a) Write down three equations that express the information given above.

(b) Find the number of each type of ticket sold.

Q8. A factory produces bags of sugar with a labelled weight of 500 g. The weights of the bags are normally distributed with a mean of 500 g and a standard deviation of 3 g.

(a) Write down the percentage of bags that weigh more than 500 g.

A bag that weighs less than 495 g is rejected by the factory for being underweight.

(b) Find the probability that a randomly chosen bag is rejected for being underweight.

A bag that weighs more than k grams is rejected by the factory for being overweight.

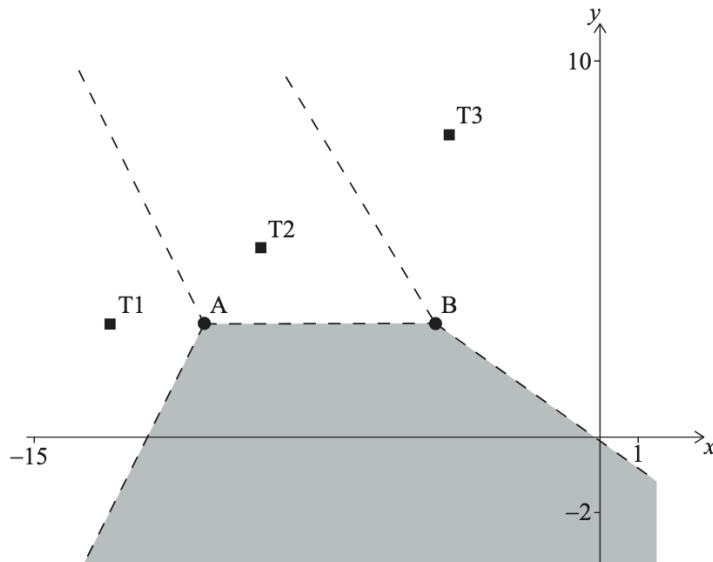
The factory rejects 2 % of bags for being overweight.

(c) Find the value of k .

Q9. The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower T4.

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower T4.

Tower T1 has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2.

Q10. Charlie and Daniella each began a fitness programme. On day one, they both ran 500m. On each subsequent day, Charlie ran 100 m more than the previous day whereas Daniella increased her distance by 2 % of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme.

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

(b) Find the value of n .

Q11. On 1 January 2022, Mina deposited \$1000 into a bank account with an annual interest rate of 4 % , compounded monthly. At the end of January, and the end of every month after that, she deposits \$100 into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places.

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than \$5000 in her account.

Q12. When the brakes of a car are fully applied the car will continue to travel some distance before it completely stops. This stopping distance, d , in metres is directly proportional to the square of the speed of the car, v , in kilometres per hour (kmh^{-1}).

When a car is travelling at a speed of $50 km h^{-1}$ it will travel 12.3 m after the brakes are fully applied before it completely stops.

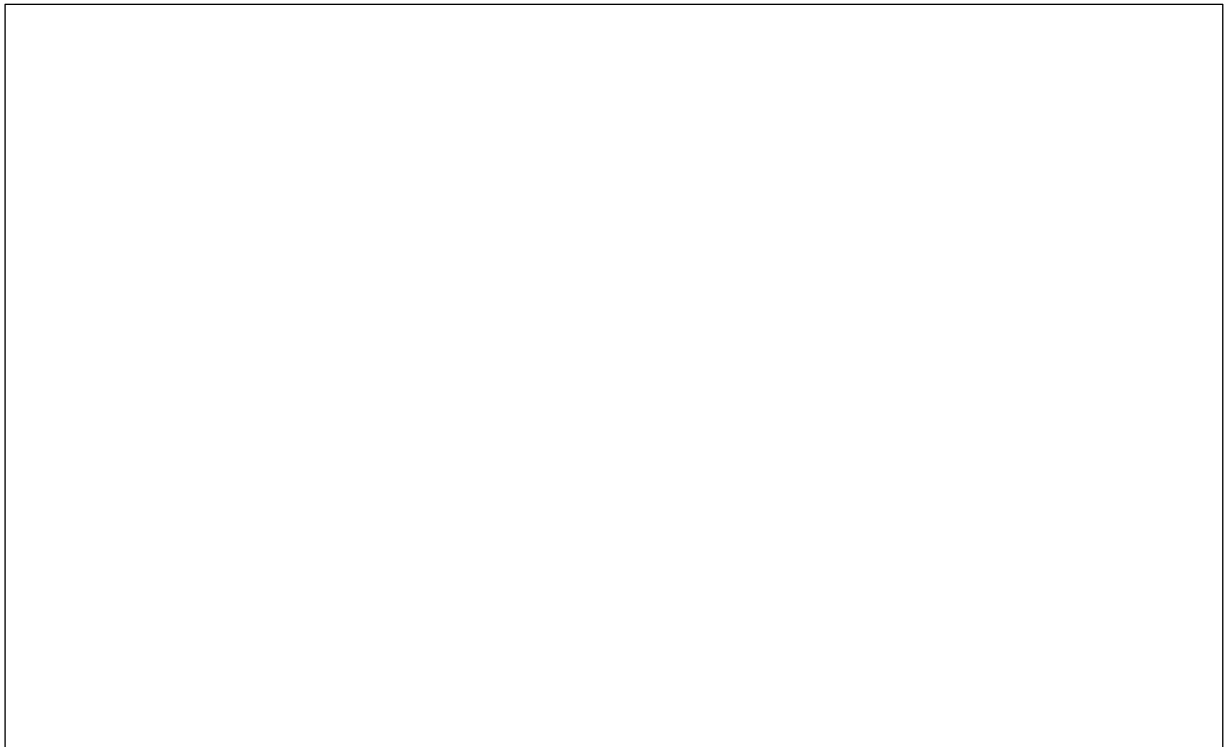
(a) Determine an equation for d in terms of v .

The police can use this equation to estimate if cars are exceeding the speed limit.

A car is found to have travelled 33 m, after fully applying its brakes, before it completely stopped.

(b) Use your equation from part (a) to estimate the speed at which this car was travelling before the brakes were applied.

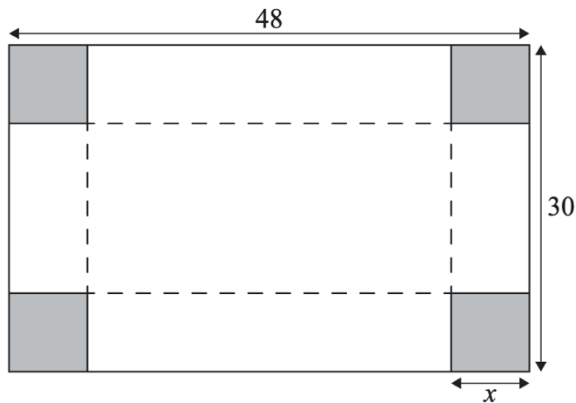
(c) After the brakes have been fully applied, identify one other variable besides speed that could affect stopping distance.



Q13. A rectangular box, with an open top, is to be constructed from a piece of cardboard that measures 48 cm by 30 cm.

Squares of equal size will be cut from the corners of the cardboard, as indicated by the shading in the diagram. The sides will then be folded along the dotted lines to form the box.

diagram not to scale



The volume of the box, in cubic centimetres, can be modelled by the function

$V(x) = (48 - 2x)(30 - 2x)(x)$, for $0 < x < k$, where x is the length of the sides of the squares removed in centimetres.

(a) Write down the maximum possible value of k in this context.

(b) Find the value of x that maximizes the volume of the box.

A second piece of 48 cm by 30 cm cardboard is damaged and a strip 2 cm wide must be removed from all four sides. A box will then be constructed in a similar manner from the remaining cardboard.

(c) Calculate the maximum possible volume of the box made from the second piece of cardboard.

Q14. The following frequency distribution table shows the test grades for a group of students.

Grade	1	2	3	4	5	6	7
Frequency	1	4	7	9	p	9	4

For this distribution, the mean grade is 4.5.

(a) Write down the total number of students in terms of p .

(b) Calculate the value of p .

Q15. Consider the graph of the following function:

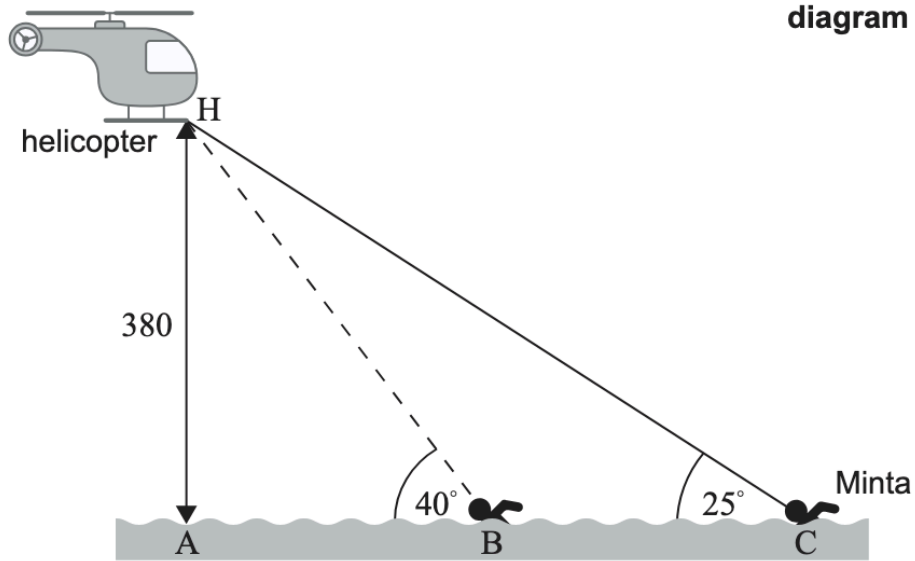
$$f(x) = \frac{16}{x} + \frac{x^2}{8}, \text{ for } x \neq 0.$$

Write down the equation of the vertical asymptote of $f(x)$.

(b) Find $f'(x)$.

(c) Write down the interval in which $f(x)$ is increasing.

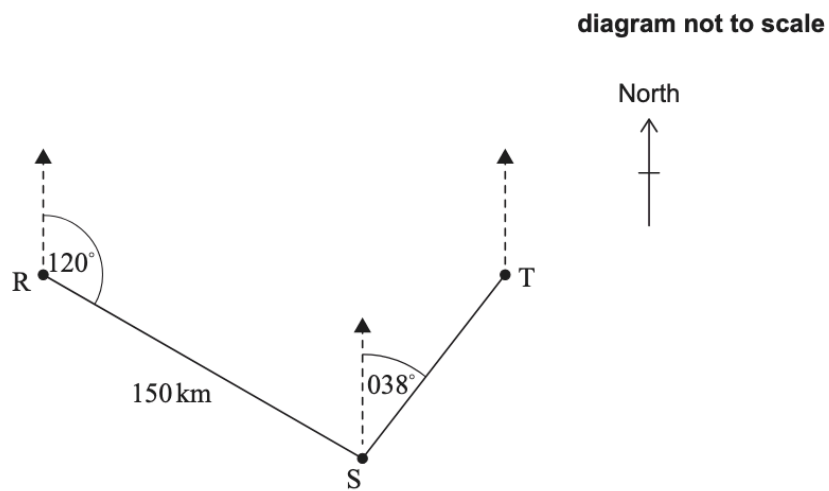
Q16. The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25° . After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40° .

- Write down the size of the angle of depression from H to C.
- Find the distance from A to C.
- Find the distance from B to C.
- Find Minta's speed, in metres per hour.

Q17. Ron sails his boat from point R for a distance of 150 km, on a bearing of 120° , to arrive at point S. He then sails on a bearing of 038° to point T. Ron's journey is shown in the diagram.



(a) Find \hat{RST} .

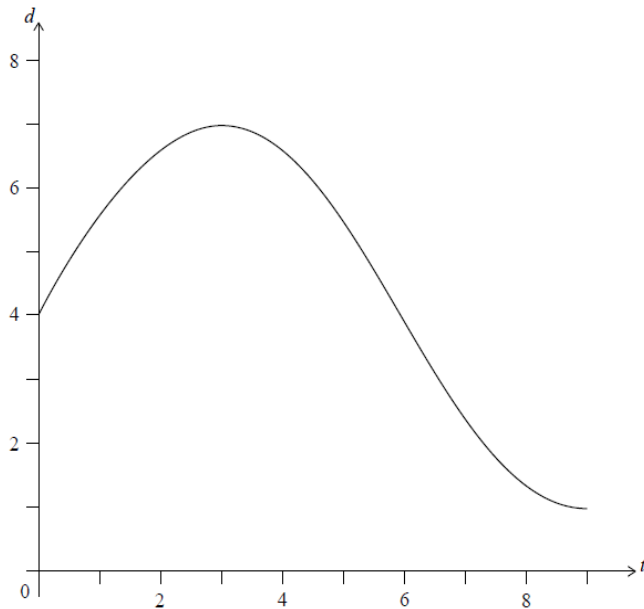
Point T is directly east of point R.

(b) Calculate the distance that Ron sails to return directly from point T to point R.

Q18. The following graph shows the depth of water, d metres, in a river at t hours after 12:00.

At 15:00, the depth of water reaches 7 m , its highest level. At 21:00, the depth of water drops to 1 m , its lowest level.

The depth can be modelled by the function $d(t) = a \sin(bt) + 4$.



(a) Find the value of a .

(b) Find the value of b .

(c) Find the first time after 12:00 when the depth is equal to 3 m . Give your answer to the nearest minute.

PAPER 2 (LONG QUESTIONS)

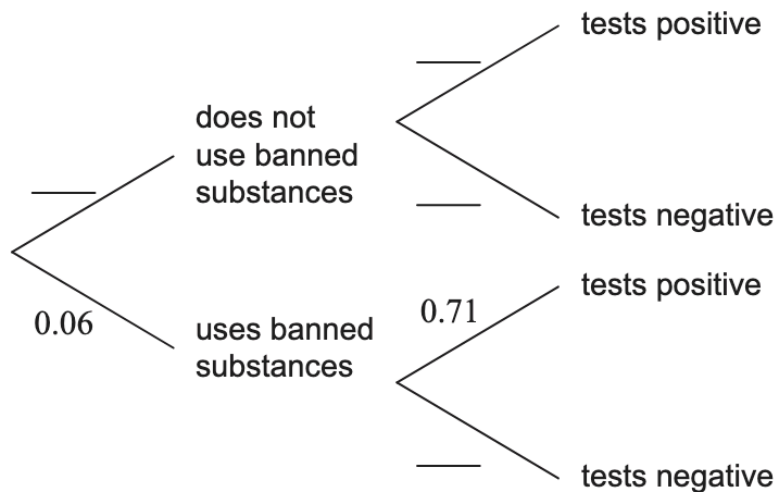
Q19. A large international sports tournament tests their athletes for banned substances. They interpret a positive test result as meaning that the athlete uses banned substances. A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06 .

If an athlete uses banned substances, the probability that they will test positive is 0.71 .

If an athlete does not use banned substances, the probability that they will test negative is 0.98 .

(a) Using the information given, copy (into your answer booklet) and complete the following tree diagram.



(b) (i) Determine the probability that a randomly selected athlete does not use banned substances and tests negative.

(ii) If two athletes are selected at random, calculate the probability that both athletes do not use banned substances and both test negative.

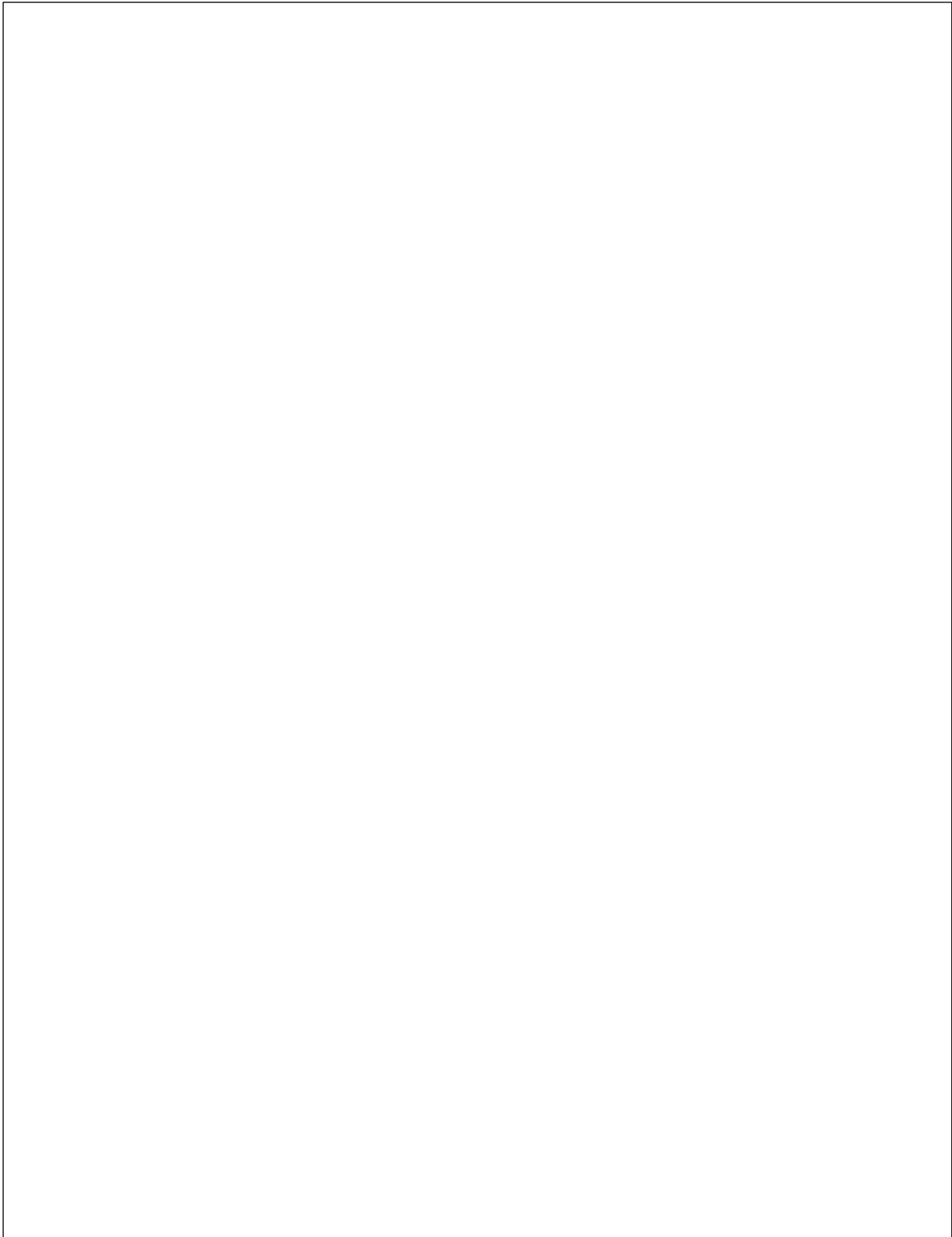
(c) (i) Calculate the probability that a randomly selected athlete will receive an incorrect test result.

(ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result.

Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

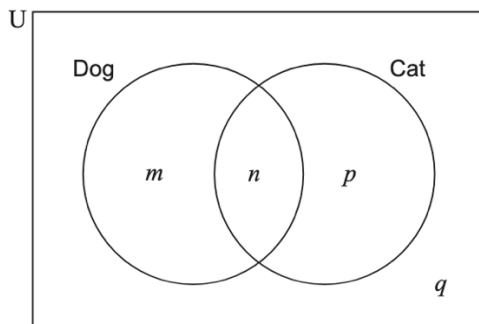
(d) Calculate the probability that none of the athletes in Team X will test positive.

(e) Determine the probability that more than 2 athletes in Team X will test positive.



Q20. At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m , n , p and q represent the percentage of students within each region.



a) Find the value of

(i) m .

(ii) n .

(iii) p .

(iv) q .

(b) Find the percentage of students who have a dog or a cat or both.

(c) Find the probability that a randomly chosen student

(i) has a dog but does not have a cat.

(ii) has a dog given that they do not have a cat.

Each year, one student is chosen randomly to be the school captain of Mirabooka

Primary School. Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events “being a school captain” and “having a dog” are independent.

Use Tim’s model to find the probability that in the next 10 years

(d) (i) 5 school captains have a dog.

(ii) more than 3 school captains have a dog.

(iii) exactly 9 school captains in succession have a dog.

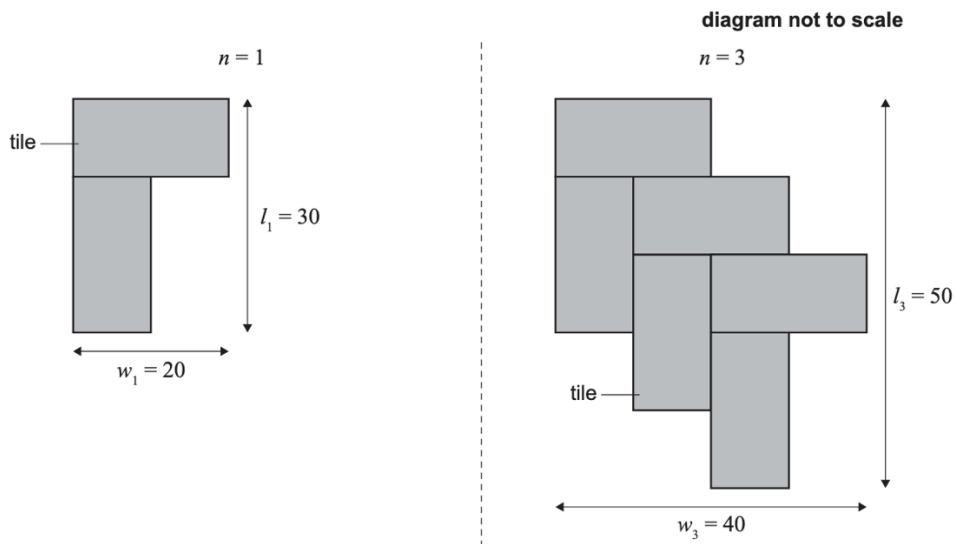
John randomly chooses 10 students from the survey.

(e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog.



Q21. Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles. Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n .

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

a) Find the value of

(i) a .

(ii) b .

(b) Write down an expression in terms of n for

(i) w_n .

(ii) l_n .

Eddie's lawn has a length 740 cm.

(c) (i) Show that Eddie needs 144 tiles.

(ii) Find the value of w for this path.

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

The tiles cost \$24.50 per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles.

To allow for breakages Eddie wants to have at least 8 % more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order.

There is a fixed delivery cost of \$35.

(g) Find the total cost for Eddie's order.

