

MARKING SCHEME 1

1. 25M.1.SL.TZ1.1

[[N/A]]

(a)

$$\ln 3 + \ln 4$$

[1]

Markscheme
$\ln 12$ A1 [1 mark]

(b)

$$3\ln 2$$

[2]

Markscheme
$\ln 2^3$ (A1) = \ln 8 A1 [2 marks]

(c)

$$-\ln \frac{1}{2}$$

[2]

Markscheme
$\ln \left(\frac{1}{2}\right)^{-1}$ OR $-(\ln 1 - \ln 2)$ OR $-\ln 2^{-1}$ (A1) = \ln 2 A1 [2 marks]

2. 25M.1.SL.TZ1.2

[5]

Markscheme	
$f'(x) = 4x^2 - 16$	A1
sets their derivative equal to zero 2 (accept $x = 2$)	(M1) $4x^2 - 16 = 0, (x = \pm 2) p =$ A1
substitutes their positive p into $f(x)$	(M1)
$y = \frac{4(2^3)}{3} - 16(2) \left(= \frac{32}{3} - 32 = -\frac{64}{3} \right)$	
$q = -\frac{64}{3}$ (accept $y = \frac{-64}{3}$)	A1
[5 marks]	

3. 25M.1.SL.TZ1.3

(a)

Write down the value of k .

[1]

Markscheme	
$k = \frac{4}{400} \left(= \frac{1}{100} = 0.01 \right)$	A1
[1 mark]	

(b)

Expand and simplify $(1 + x)^4$.

[2]

Markscheme	
attempt to find binomial coefficients or multiply out brackets Pascal's triangle down to correct row	(M1) e.g. substitute
OR $(1 + 2x + x^2)^2$ into binomial expansion $(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$	OR A1

[2 marks]

(c)

Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar.

[4]

Markscheme

METHOD 1 recognition that the expansion can be used with x replaced

with k **(M1)** $\left(1 + \frac{1}{100}\right)^4 = 1 + \frac{4}{100} + \frac{6}{100^2} + \dots (= 1 + 0.04 + 0.0006 + \dots)$ **(A1)**

multiplies by 1 000 (seen anywhere) **(M1)** $1\,000 \left(1 + \frac{1}{100}\right)^4$ $1\,000 + 40 + 0.6 + \dots (= 1\,040.6 \dots) = 1\,041$ (dinar) **A1** **METHOD 2** attempt to find the value of $(1 + k)^4$ by hand **(M1)** $(1.01)^4 = (1.0201)(1.01)^2 = (1.030301)(1.01) = 1.0406 \dots$ **(A1)**

multiplies by 1 000 (seen anywhere) **(M1)** $1\,000(1.01)^4 = 1\,040.6 \dots = 1\,041$ (dinar) **A1**
[4 marks]

4. 25M.1.SL.TZ1.4

[4]

Markscheme

METHOD 1 attempt to set up integral $e^x - (-e^x) = 2e^x$ or $2e^x$ and then

double **(M1)** $\int (e^x - (-e^x)) dx$ **OR** $2 \int e^x dx = 2 \int_{-1}^1 e^x dx = 2[e^x]_{-1}^1$ **(A1)** attempt to substitute correct limits into their integrated function and subtract **(M1)** $= 2 \left(e - \frac{1}{e}\right)$, $2e - \frac{2}{e}$, $2e -$

$2e^{-1}$ **A1** **METHOD 2** $\int_{-1}^1 e^x dx = [e^x]_{-1}^1$ and $\int_{-1}^1 -e^x dx = [-e^x]_{-1}^1$ **(A1)** attempt to substitute correct limits into both their integrated functions and subtract **(M1)** $e^1 - e^{-1}$ and $-e^1 - (-e^{-1})$ subtracts their two integrals in correct order **(M1)** $e^1 - e^{-1} - (-e^1 + e^{-1}) = 2 \left(e - \frac{1}{e}\right)$, $2e - \frac{2}{e}$, $2e - 2e^{-1}$ **A1**

[4 marks]

5. 25M.1.SL.TZ1.5

(a)

Find $P(A \cap B)$.

[3]

Markscheme

$$P(A) = \frac{1}{4} \quad (\mathbf{A1}) \text{ attempt to use } P(B|A) = \frac{P(B \cap A)}{P(A)} \quad (\mathbf{M1}) \frac{2}{3} = \frac{P(B \cap A)}{\left(\frac{1}{4}\right)}$$
$$P(A \cap B) = \frac{2}{3} \left(\frac{1}{4}\right) = \frac{2}{12} \left(= \frac{1}{6}\right) \quad \mathbf{A1}$$

[3 marks]

(b)

Show that events A and B are independent.

[3]

Markscheme

attempt to use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **OR** a Venn diagram, with their values of $P(A)$ and $P(B \cap A)$ **M1** $\frac{3}{4} = \frac{1}{4} + P(B) - \frac{1}{6}$

$$P(B) = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} \left(= \frac{2}{3}\right) \quad \mathbf{A1} P(B|A) = P(B) \quad \mathbf{OR} \quad P(A)P(B) =$$
$$\frac{1}{6} \text{ so } P(A \cap B) = P(A)P(B) \text{ (hence } A \text{ and } B \text{ are independent)}$$

R1 Note: The **R1** is dependent on all previous marks.

[3 marks]

6. 25M.1.SL.TZ2.1

(a)

Find $f'(1)$.

[2]

Markscheme

attempt to differentiate (at least one correct term seen) **(M1)** $f'(x) = 3x^2 + 10x + 3 + 10$ **A1** $f'(1) = 13$
[2 marks]

(b)

Find the equation of the tangent to the graph of f at $x = 1$.

[2]

Markscheme

attempt to find $f(1)$ **(M1)** $1 + 5 - 8 = -2$ $y + 2 = 13(x - 1)$ **A1**
[2 marks]

7. 25M.1.SL.TZ2.2

[5]

Markscheme

recognises to integrate $g'(x)$ **(M1)** $\int g'(x)dx$ **OR** $\int (\cos x + e^{2x})dx$
 $g(x) = \sin x + \frac{1}{2}e^{2x} + C$ **A1A1**

substitutes (0,7) into their integrated expression (must involve $+C$) **(M1)** $g(0) = 0 + \frac{1}{2} + C = 7 \Rightarrow C = 6.5$ $g(x) = \sin x + \frac{1}{2}e^{2x} + 6.5$ **A1**
[5 marks]

8. 25M.1.SL.TZ2.3

[[N/A]]

(a)

Write down the value of $f(0)$.

[1]

Markscheme	
-2 (accept $(0, -2)$) [1 mark]	A1

(b)

Write down the equation of the horizontal asymptote.

[1]

Markscheme	
$y = \frac{3}{2}$ (must be an equation) Note: Do not accept \neq sign. [1 mark]	A1

(c)

Find the range of g .

[3]

Markscheme	
attempt to reflect the graph of f in the x -axis OR one correct value seen (M1) both correct values $-\frac{3}{2}$ and 2 (seen anywhere) A1 $-\frac{3}{2} < y \leq 2$ A1 [3 marks]	

9. 25M.1.SL.TZ2.4

(a)

Using the sine rule, show that $\cos \theta = \frac{3\sqrt{2}}{5}$.

[3]

Markscheme

correct substitution in sine rule **(A1)** $\frac{\sin \theta}{5} = \frac{\sin 2\theta}{6\sqrt{2}}$ (or equivalent)
 attempt to use double angle rule for $\sin 2\theta$ **(M1)** $\frac{\sin \theta}{5} = \frac{2 \sin \theta \cos \theta}{6\sqrt{2}}$
 $6\sqrt{2} \sin \theta = 10 \sin \theta \cos \theta$ **OR** $\frac{1}{5} = \frac{2 \cos \theta}{6\sqrt{2}}$ **OR** equivalent **A1** $\cos \theta =$
 $\frac{3\sqrt{2}}{5}$ **AG**
[3 marks]

(b)

Hence, find $\sin \theta$.

[2]

Markscheme

valid attempt to find $\sin \theta$ **(M1)** $\sin^2 \theta + \left(\frac{3\sqrt{2}}{5}\right)^2 = 1$ **OR** right triangle
 with adjacent side and hypotenuse labelled $\sin \theta = \frac{\sqrt{7}}{5}$ **A1**
[2 marks]

(c)

Find DC .

[2]

Markscheme

$\frac{1}{2} \times 6\sqrt{2} \times DC \times \frac{\sqrt{7}}{5} = 2\sqrt{14}$ **(A1)** $DC = \frac{10}{3}$ **A1**
[2 marks]

10. 25M.1.SL.TZ3.3

(a)

Write down the value of $f(-3)$.

[1]

Markscheme
$f(-3) = -1$ A1 [1 mark]

(b)

State the domain of f^{-1} , the inverse function of f .

[1]

Markscheme
$-3 \leq x \leq 5$ A1 Note: Award A1 for answers using interval notation $[-3, 5]$. [1 mark]

(c)

Find the value of x that satisfies $f^{-1}(2x - 7) = -3$.

[3]

Markscheme
$(f^{-1}(2x - 7) = -3 \Rightarrow) 2x - 7 = f(-3)$ OR $f^{-1}(-1) = -3$ (M1) $2x - 7 = -1$ (A1) $x = 3$ A1 [3 marks]

11. 24N.1.SL.TZ1.1

(a)

Write down

[[N/A]]

(a.i)

the mode;

[1]

Markscheme	
27	A1 [1 mark]

(a.ii)

the range;

[1]

Markscheme	
30	A1 [1 mark]

(a.iii)

the median.

[1]

Markscheme	
35	A1 [1 mark]

(b)

Find the interquartile range.

[3]

Markscheme

$Q_1 = 31$ and $Q_3 = 46$ (A1) attempt to subtract their upper and lower quartiles (M1) Note: Award M1 only for correct values, or for values clearly indicated as candidate's Q_1 and Q_3 . $46 - 31$ IQR = 15 A1
[3 marks]

12. 24N.1.SL.TZ1.2

(a)

Find the value of r .

[3]

Markscheme

$\frac{1}{2}r^2(\theta) = 6$ OR $\frac{1}{2}r^2(0.75) = 6$ (A1) attempt to solve their equation to find r or r^2 (M1) Note: To award the M1, candidate's equation must include r^2 and $\theta = 1.5$, and they must isolate r^2 or r . $r^2 = 16$ $r = 4$ (cm) A1
[3 marks]

(b)

Hence, find the perimeter of sector OAB.

[2]

Markscheme

evidence of summing the two radii and the arc length (M1)
perimeter = $2r + r\theta = 8 + 4(0.75) = 11$ (cm) A1
[2 marks]

13. 24N.1.SL.TZ1.4

[4]

Markscheme

METHOD 1 attempt to expand $(2n + 5)^2 - (2n - 5)^2$ **M1 Note:**
Award **M0** for invalid attempts such as $(2n + 5)^2 = 4n^2 + 25$. $= 4n^2 + 20n + 25 - (4n^2 - 20n + 25)$ or equivalent **A1** = 40n OR $20n + 20n$ **A1** = 10(4n) OR $\frac{40}{10} = 4$ OR $\frac{40n}{4} = 10n$ OR $20n + 20n = 10(2n + 2n)$ (or equivalent) **R1** so is a multiple of 10 **AG Note:** Do not award the **R1** unless both **A** marks have been awarded. **METHOD 2** use of $a^2 - b^2 = (a + b)(a - b)$ where $a = 2n + 5$, $b = 2n - 5$ **M1** = $(2n + 5 + 2n - 5)(2n + 5 - 2n + 5)$ **A1A1 Note:** Award **A1** for each correct bracket. $= 4n \times 10$ (= 40n) **R1** so is a multiple of 10 **AG Note:** Do not award the **R1** unless both **A** marks have been awarded.
[4 marks]

14. 24M.1.SL.TZ1.1

(a)

Find the value of the common difference.

[2]

Markscheme

valid method to find the common difference **(M1)** $d = \frac{22-10}{2}$ OR $10 + 2d = 22$ OR $u_1 + d = 10$, $u_1 + 3d = 22$ OR $u_3 = 16$
 $d = 6$ **A1**
[2 marks]

(b)

Find an expression for u_n , the n th term.

[2]

Markscheme

$u_1 = 10 - 6 (= 4)$ **(A1)** $u_n = 4 + 6(n - 1)$ OR $u_n = 6n - 2$ **A1**
[2 marks]

15. 24M.1.SL.TZ1.2

(a)

Find the value of p and the value of q .

[5]

Markscheme

attempt to form equation for the sum of frequencies = 16 or mean =
3 **(M1)** $p + q + 4 + 2 + 3 = 16 (\Rightarrow p + q = 7)$ **A1** $\frac{p+2q+12+8+18}{16} =$
 $3 (\Rightarrow p + 2q = 10)$ OR $\frac{p+2q+12+8+18}{9+p+q} = 3 (\Rightarrow 2p + q = 11)$ **A1** attempt
to eliminate one variable from their equations **(M1)** $p + 2(7 - p) +$
 $38 = 48$ OR $2(7 - q) + q = 11$ $p = 4$ and $q = 3$ **A1** **Note:** Award
M1A0A0M0A1 for $p = 4, q = 3$ with no working.
[5 marks]

(b)

Write down the mean final score.

[1]

Markscheme

mean final score = 30 **A1**
[1 mark]

16. 24M.1.SL.TZ2.1

(a)

Write down the value of

[[N/A]]

(a.i)

$f(4)$;

[1]

Markscheme	
$f(4) = 1$ [1 mark]	A1

(a.ii)

$$f \circ f(4);$$

[1]

Markscheme	
$f \circ f(4) = 3$ [1 mark]	A1

(a.iii)

$$f^{-1}(3).$$

[1]

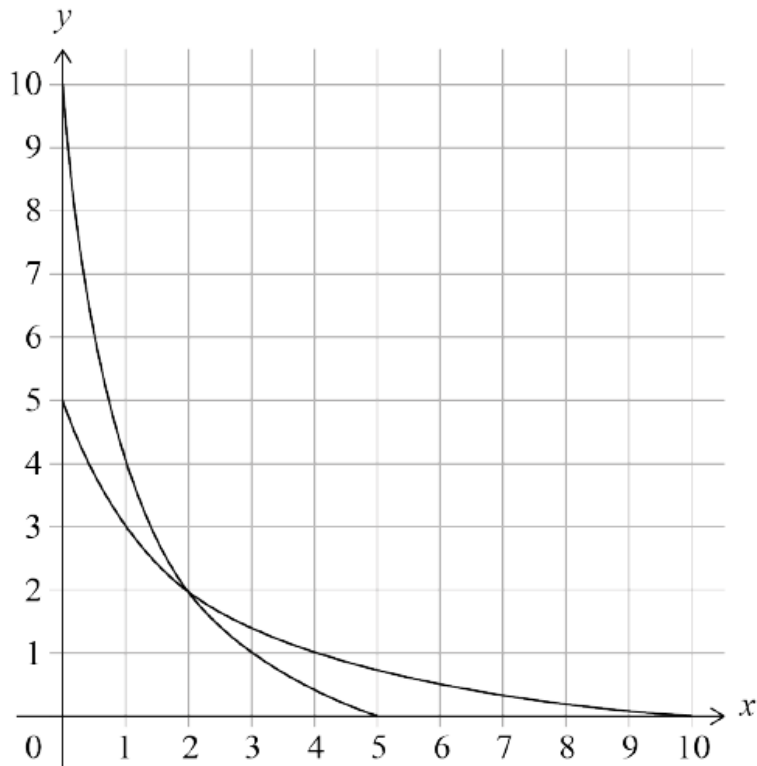
Markscheme	
$f^{-1}(3) = 1$ [1 mark]	A1

(b)

On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes.

[2]

Markscheme



curve with y intercept at $(0, 10)$ and x intercept at $(5, 0)$
 passes through $(2, 2)$ OR through $(1, 4)$ and $(3, 1)$

concave up
A1 curve
A1 Note: Do not award the second mark unless the first mark has been awarded. (Do not award **A0A1**).

[2 marks]

17. 24M.1.SL.TZ2.3

(a)

Solve $3m^2 + 5m - 2 = 0$.

[3]

Markscheme

valid attempt to solve a quadratic equation (factorising, use of formula, completing square, or otherwise) **(M1)** $(3m - 1)(m + 2) =$

0 OR $m = \frac{-5 \pm \sqrt{25 + 24}}{6}$ (or equivalent) **(A1)** $m = \frac{1}{3}, m = -2$ **A1**

[3 marks]

(b)

Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$.

[2]

Markscheme

setting their m -value(s) = 3^x OR recognising a quadratic in 3^x **(M1)**
 $3^x = \frac{1}{3}$ (or $3^x = -2$) OR $3 \times (3^x)^2 + 5 \times 3^x - 2 = 0$ $x =$
 -1 **A1 Note:** Award the final **A1** if candidate's answer includes $x =$
 -1 and $x = \log_3(-2)$. Award **A0** if other incorrect answers are given.
[2 marks]

18. 24M.1.SL.TZ2.5

(a)

Complete the tree diagram to show the probabilities of not nesting in each season.

Write your answers in terms of k .

[2]

Markscheme

(b.ii)

Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution.

[1]

Markscheme

$(k = \frac{1}{3}$ is the only valid solution as) $\frac{8}{3} > 1$ **R1 Note:** Accept any valid reasoning indicating that any probability cannot be greater than 1 and/or probability cannot be less than 0.
[1 mark]

19. 23N.1.SL.TZ1.1

[[N/A]]

(a)

Write down the value of a .

[1]

Markscheme

$a = 7$ **A1**
[1 mark]

(b.i)

Write down the period of f .

[1]

Markscheme

period = π
[1 marks]

A1

(b.ii)

Hence, find the value of b .

[2]

Markscheme

$b = \frac{2\pi}{\pi}$ OR $\pi = \frac{2\pi}{b}$
[2 marks]

(A1) = 2

A1

(c)

Find the value of $f\left(\frac{\pi}{12}\right)$.

[3]

Markscheme

substituting $\frac{\pi}{12}$ into their $f(x)$
 $\frac{1}{2}$
[3 marks]

(A1) = $\frac{7}{2}$

A1

(M1) $f\left(\frac{\pi}{12}\right) = 7 \sin\left(\frac{\pi}{6}\right) \sin\left(\frac{\pi}{6}\right) =$

20. 23N.1.SL.TZ1.3

[[N/A]]

(a)

Find $P(A \cup B)$.

[2]

Markscheme

$$(P(A \cup B) =) 0.7 + 0.75 - 0.55$$

[2 marks]

$$(A1) = 0.9$$

A1

(b)

Hence, otherwise find $P(A' \cap B')$.

[2]

Markscheme

recognition that $A' \cap B' = (A \cup B)'$ OR $A' \cap B' = 1 - A \cup B$ (region/value may be seen in a correctly shaded/labeled Venn diagram) **(M1)**
(= $1 - 0.9$) = 0.1 **A1 Note:** For the final mark, 0.1 must be stated as the candidate's answer, or labeled as $P(A' \cap B')$ in their Venn diagram. Just seeing an unlabeled 0.1 in the correct region of their diagram earns **M1A0**.
[2 marks]

21. 23M.1.SL.TZ1.1

(a)

Find the coordinates of M .

[2]

Markscheme

$M (6, -3)$ **A1A1 [2 marks]**

(b)

Find the gradient of L .

[2]

Markscheme

$$\text{gradient of } [PQ] = -\frac{5}{9} \quad (\mathbf{A1}) \text{ gradient of } L = \frac{9}{5} \quad \mathbf{A1 [2 marks]}$$

(c)

Hence, write down the equation of L .

[1]

Markscheme

$$y + 3 = \frac{9}{5}(x - 6) \text{ OR } y = \frac{9}{5}x - \frac{69}{5} \text{ (or equivalent)} \quad \mathbf{A1 \text{ Note: Do not}} \\ \text{accept } L = \frac{9}{5}x - \frac{69}{5}. \quad \mathbf{[1 mark]}$$

22. 23M.1.SL.TZ1.2

[[N/A]]

(a)

Find the zero of $f(x)$.

[2]

Markscheme

$$\text{recognizing } f(x) = 0 \quad (\mathbf{M1}) x = -1 \quad \mathbf{A1 [2 marks]}$$

(b)

For the graph of $y = f(x)$, write down the equation of

[[N/A]]

(b.i)

the vertical asymptote;

[1]

Markscheme

$x = 2$ (must be an equation with x) **A1 [1 mark]**

(b.ii)

the horizontal asymptote.

[1]

Markscheme

$y = \frac{7}{2}$ (must be an equation with y) **A1 [1 mark]**

(c)

Find $f^{-1}(x)$, the inverse function of $f(x)$.

[3]

Markscheme

EITHER interchanging x and y **(M1)** $2xy - 4x = 7y + 7$ correct working with y terms on the same side: $2xy - 7y = 4x + 7$ **(A1)** **OR** $2yx - 4y = 7x + 7$ correct working with x terms on the same side: $2yx - 7x = 4y + 7$ **(A1)** interchanging x and y **OR** making x the subject $x = \frac{4y+7}{2y-7}$ **(M1)** **THEN** $f^{-1}(x) = \frac{4x+7}{2x-7}$ (or equivalent) $(x \neq \frac{7}{2})$ **A1 [3 marks]**

23. 23M.1.SL.TZ1.4

(a)

Show that the equation $\cos 2x = \sin x$ can be written in the form $2 \sin^2 x + \sin x - 1 = 0$.

[1]

Markscheme

$1 - 2 \sin^2 x = \sin x$ **A1** $2 \sin^2 x + \sin x - 1 = 0$ **AG [1 mark]**

(b)

Hence, solve $\cos 2x = \sin x$, where $-\pi \leq x \leq \pi$.

[5]

Markscheme

valid attempt to solve quadratic **(M1)** $(2 \sin x - 1)(\sin x + 1)$ OR
 $\frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)}$ recognition to solve for $\sin x$ **(M1)** $\sin x = \frac{1}{2}$ OR $\sin x =$
 -1 any correct solution from $\sin x = -1$ **A1**
 any correct solution from $\sin x = \frac{1}{2}$ **A1**

Note: The previous two marks may be awarded for degree or radian values, irrespective of domain.

$x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ **A1**

Note: If no working shown, award no marks for a final value(s).

Award **A0** for $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ if additional values also given. **[5 marks]**

24. 23M.1.SL.TZ1.5

[6]

Markscheme

recognition of quadratic in e^x **(M1)** $(e^x)^2 - 3e^x + \ln k (= 0)$ OR $A^2 -$
 $3A + \ln k (= 0)$ recognizing discriminant ≥ 0 (seen anywhere) **(M1)**
 $(-3)^2 - 4(1)(\ln k)$ OR $9 - 4 \ln k$ **(A1)** $\ln k \leq \frac{9}{4}$ **(A1)** $e^{9/4}$ (seen
 anywhere) **A1** $0 < k \leq e^{9/4}$ **A1 [6 marks]**

25. 22M.1.SL.TZ1.2

(a)

The expression $\frac{3\sqrt{x}-5}{\sqrt{x}}$ can be written as $3 - 5x^p$. Write down the value of p .

[1]

Markscheme

$$\frac{3\sqrt{x}-5}{\sqrt{x}} = 3 - 5x^{-\frac{1}{2}} \quad \mathbf{A1}$$

$$p = -\frac{1}{2}$$

[1 mark]

(b)

Hence, find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}} \right) dx$.

[4]

Markscheme

$$\int \frac{3\sqrt{x}-5}{\sqrt{x}} dx = 3x - 10x^{\frac{1}{2}}(+c) \quad \mathbf{A1A1}$$

substituting limits into their integrated function and subtracting **(M1)**

$$3(9) - 10(9)^{\frac{1}{2}} - \left(3(1) - 10(1)^{\frac{1}{2}} \right) \text{ OR } 27 - 10 \times 3 - (3 - 10)$$

$$= 4 \quad \mathbf{A1}$$

[4 marks]

26. 22M.1.SL.TZ1.3

(a)

Find the largest value of c that would not be considered an outlier.

[3]

Markscheme

$$\begin{aligned} \text{IQR} &= 10 - 6 (= 4) && \text{(A1)} \\ \text{attempt to find } Q_3 + 1.5 \times \text{IQR} &&& \text{(M1)} \\ & && 10 + 6 \\ 16 && \text{A1} \end{aligned}$$

[3 marks]

(b.i)

One of the adults surveyed is 42 years old. Estimate the age of their eldest child.

[2]

Markscheme

$$\begin{aligned} \text{choosing } c &= \frac{1}{2}a - 9 && \text{(M1)} \\ & && \frac{1}{2} \times 42 - 9 \\ = 12 \text{ (years old)} && \text{A1} \end{aligned}$$

[2 marks]

(b.ii)

Find the mean age of all the adults surveyed.

[2]

Markscheme

$$\begin{aligned} \text{attempt to solve system by substitution or elimination} &&& \text{(M1)} \\ 34 \text{ (years old)} && \text{A1} \end{aligned}$$

[2 marks]

27. 22M.1.SL.TZ1.5

[5]

Markscheme

evidence of using product rule **(M1)**
 $\frac{dy}{dx} = (2x - 1) \times (ke^{kx}) + 2 \times e^{kx} \quad (= e^{kx}(2kx - k + 2))$ **A1**
 correct working for one of (seen anywhere) **A1**

$$\frac{dy}{dx} \text{ at } x = 1 \Rightarrow ke^k + 2e^k$$

OR

slope of tangent is $5e^k$

their $\frac{dy}{dx}$ at $x = 1$ equals the slope of $y = 5e^kx$ ($= 5e^k$) (seen anywhere) **(M1)**

$$ke^k + 2e^k = 5e^k$$

$$k = 3 \quad \mathbf{A1}$$

[5 marks]

28. 25M.2.SL.TZ1.1

(a)

Show that $\frac{(x-1)^2}{x} = x - 2 + \frac{1}{x}$.

[2]

Markscheme

METHOD 1 attempt to expand brackets on the numerator **M1**
 $(x - 1)^2 = x^2 - 2x + 1 \quad \frac{(x-1)^2}{x} = \frac{x^2-2x+1}{x} \quad (= \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x})$ **A1** $= x - 2 +$

$\frac{1}{x}$ **AG** **METHOD 2** attempt to express terms with a common

denominator **M1** $x - 2 + \frac{1}{x} = \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x} = \frac{x^2-2x+1}{x}$ **A1** $=$

$$\frac{(x-1)^2}{x} \quad \mathbf{AG}$$

[2 marks]

(b)

Hence, find $\int f(x) dx$.

[3]

Markscheme

$$\int f(x) dx = \frac{x^2}{2} - 2x + \ln|x|(+c) \quad \mathbf{A1A1A1}$$

Note: Award **A1** for each correct term.
[3 marks]

29. 25M.2.SL.TZ1.2

(a)

the area of the sector BOA ;

[3]

Markscheme

EITHER $210^\circ = \frac{210\pi}{180} (= \frac{7\pi}{6} = 3.66519\dots)$ radians **(A1)** attempt to use
 radian formula for area of sector **(M1)** area = $\frac{1}{2}(19.5)^2 \left(\frac{7\pi}{6}\right)$

OR attempt to use degree formula for area of sector **(M1)** area =
 $\frac{210}{360}\pi(19.5)^2$ **(A1)**

THEN area = $\frac{3549\pi}{16} = 696.844\dots = 697 \left(= \frac{3549\pi}{16}\right) (cm^2)$ **A1**
[3 marks]

(b)

the radius of the cone.

[3]

Markscheme

EITHER arc length = $19.5 \left(\frac{7\pi}{6}\right)$ **OR** = $\frac{210}{360}(2\pi(19.5)) \left(= \frac{91\pi}{4} = 71.4712\dots\right)$ **(A1)** attempt to set $2\pi r$ equal to arc length **(M1)**
 $2\pi r = 71.4712\dots$ **OR** attempt to set $\pi r l$ equal to their area from

(a) (M1) $19.5\pi r = 696.844\dots$ (A1) THEN $r = 11.4$ ($= \frac{91}{8} = 11.375$) (cm) A1
[3 marks]

30. 25M.2.SL.TZ1.3

(a)

Write down the period of f .

[1]

Markscheme

period is $\frac{\pi}{2}$ ($= 1.57079\dots = 1.57$) A1
[1 mark]

(b)

Find the value of a and the value of b .

[3]

Markscheme

attempt to substitute $x = \frac{\pi}{12}$, $f(x) = 5$ and $x = \frac{\pi}{3}$, $f(x) = 7$ to obtain two equations (M1)

Note: accept work where x values have been converted into degrees

$a \tan\left(\frac{\pi}{6}\right) + b = 5$ and $a \tan\left(\frac{2\pi}{3}\right) + b = 7$ ($\Rightarrow \frac{a}{\sqrt{3}} + b = 5$ and $-a\sqrt{3} + b = 7$)
 $a = -\frac{\sqrt{3}}{2}$ ($= -0.866025\dots = -0.866$) A1 $b = \frac{11}{2}$ ($= 5.5$) A1

Note: These A1 marks may be awarded independently.
[3 marks]

31. 25M.2.SL.TZ1.4

Markscheme

METHOD 1 attempt to find change in population using a definite integral **(M1)** $t = 4$ at the start of 2026 (seen anywhere) **(A1)**
 $\int_0^4 -104\,000e^{-0.0145t} dt$ **(A1)** = -404165.8... **(A1)**

attempt to add initial population to their change in population from a definite integral **(M1)** population at the start of 2026 = $6.78 \times 10^6 - 404\,165.8... = 6\,375\,834.1... = 6\,380\,000 (= 6.38 \times 10^6)$ **A1 METHOD**

2 attempt to find population using an indefinite integral **(M1)** $P = \int -104\,000e^{-0.0145t} dt = \frac{-104\,000e^{-0.0145t}}{-0.0145} + c (= 7\,172\,413.7...e^{-0.0145t} + c)$ **(A1)**

attempt to substitute $t = 0$, $P = 6.78 \times 10^6$ into equation with c . **(M1)**
 $6.78 \times 10^6 = 7\,172\,413.7... + c \Rightarrow c = -392\,413.7... P = 7\,172\,413.7...e^{-0.0145t} - 392\,413.7...$ **(A1)**

$t = 4$ at the start of 2026 (seen anywhere) **(A1)**
 population at the start of 2026 = $7\,172\,413.7...e^{-0.0145(4)} - 392\,413.7... = 6\,375\,834.1... = 6\,380\,000 (= 6.38 \times 10^6)$ **A1**
[6 marks]

32. 24N.2.SL.TZ1.1

[[N/A]]

(a)

Find the value of

[[N/A]]

(a.i)

 $f(0)$;

[1]

Markscheme

$$f(0) = -7 \quad \mathbf{A1}$$

[1 mark]

(a.ii)

$f(60)$.

[1]

Markscheme

$-12.7782 \dots f(60) = -12.8 \quad (= 7\sqrt{60} - 67 = 14\sqrt{15} - 67) \quad \mathbf{A1}$
[1 mark]

(b)

Find the two roots of $f(x) = 0$.

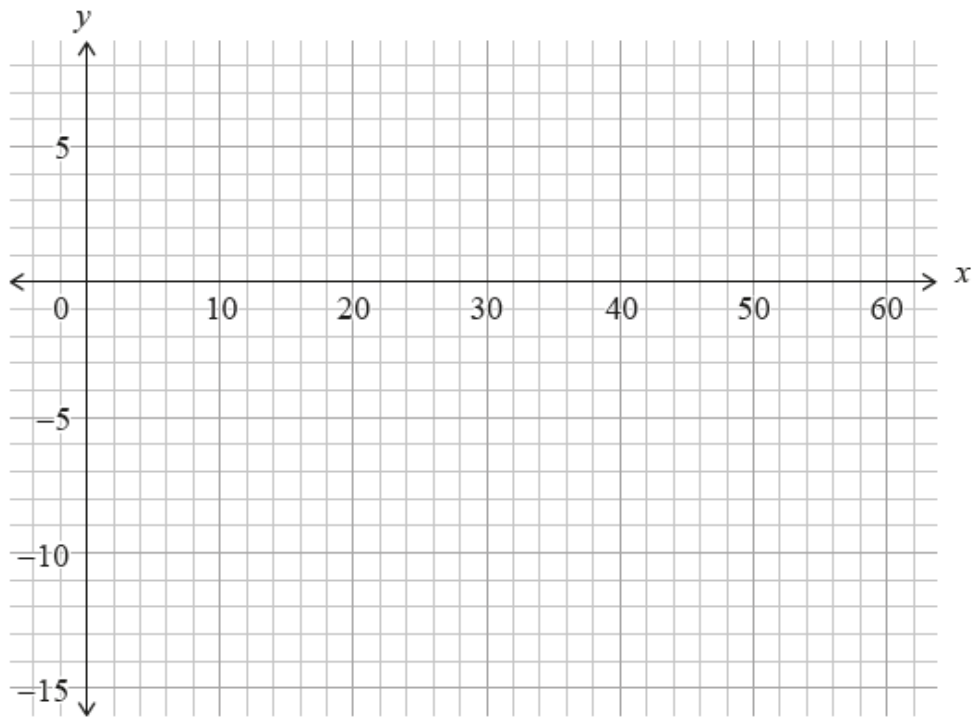
[2]

Markscheme

attempt to find at least one root $\mathbf{(M1)}$ $x = 1.46098 \dots$ and $x =$
 $33.5390 \dots$ $x = 1.46$ and $x = 33.5 \quad \mathbf{A1}$
[2 marks]

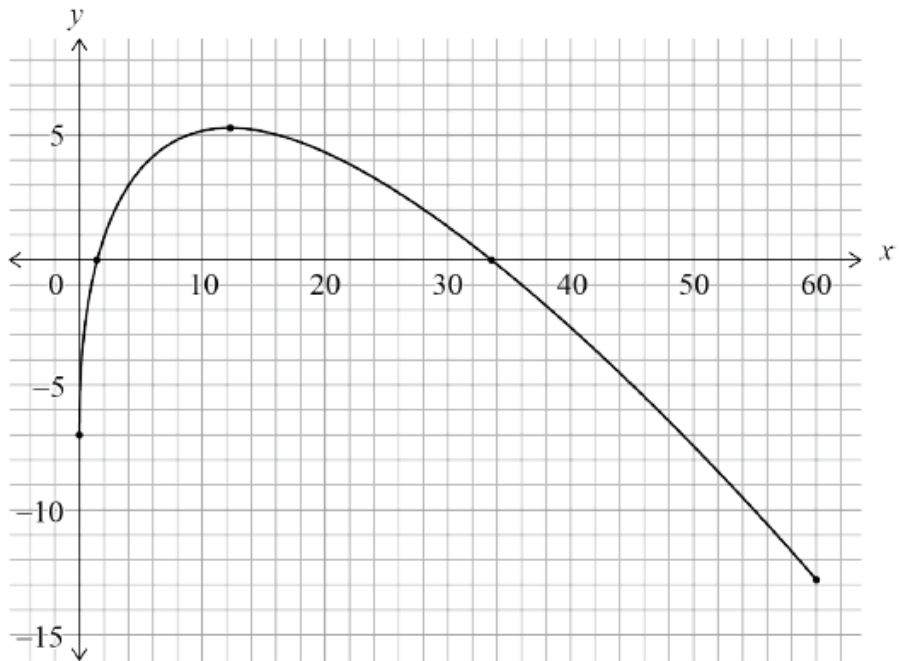
(c)

Sketch the graph of $y = f(x)$ on the following grid.



[3]

Markscheme



A1A1A1

Note: Follow through from their part (a). Award **A1** for endpoints at approximately (0, 7) and (60, -12.8). Allow for y-intercept $-8 < y < -6$ and for the right endpoint in the interval $x = 60$, $-13 < y < -12$. The following two **A** marks may only be awarded if the approximate shape is correct. Award **A1** for x-intercepts at approximately $x = 1.5$ and $x = 33.5$ and award **A1** for maximum at approximately (12.2, 5.3). Allow for x-intercepts in the intervals $0 < x < 2$, $32 < x < 34$, and maximum in the intervals $10 < x < 14$, $4.5 < y < 6$.

[3 marks]

33. 24N.2.SL.TZ1.2

[4]

Markscheme

EITHER attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen **(M1)**
 $(2x)^8(-5)^3$ OR $(2x)^8(-5)^3$ OR $165 \times (2x)^8(-5)^3$. **(A1)(A1)** **Note:**
 Award **A1** for or or 165, **A1** for $(2x)^8(-5)^3$. **OR** attempt to use the
 general term **(M1)** $(2x)^{11-r}(-5)^r$ and $r = 3$ **(A1)(A1)** **THEN**
 -5280000 (exact) **A1** **Note:** Award **A0** for a final answer of
 $-5280000x^8$.

[4 marks]

34. 24N.2.SL.TZ1.4

(a)

Find the value of k .

[2]

Markscheme

recognition of sum of probabilities equals 1 **(M1)** $\frac{2k}{15} + \frac{4k}{15} + \frac{7k}{15} + \frac{10k}{15} = 1$

$k = 0.652173 \dots$ $k = 0.652$ $\left(= \frac{15}{23} \right)$ **A1**

[2 marks]

(b)

Find $E(X)$.

[3]

Markscheme

correct probabilities:

$$\frac{2}{23}, \frac{4}{23}, \frac{7}{23}, \frac{10}{23} \text{ OR } 0.0870, 0.174, 0.304, 0.435$$

(A1) substitution of their

probabilities into formula for expected value

$$\mathbf{(M1)} 2 \times \frac{2}{23} + 4 \times \frac{4}{23} +$$

$$7 \times \frac{7}{23} + 10 \times \frac{10}{23} \text{ OR } \frac{169k}{15} = 7.34782 \dots E(X) = 7.35 \left(= \frac{169}{23} \right) \text{ (same 3sf}$$

from previous 3sf answer) **A1**

[3 marks]

35. 25M.2.SL.TZ3.4

(a.i)

Find the value of a .

[2]

Markscheme

summing probabilities and equating to 1
 $0.026 = 1$ ($a =$) 0.154 **A1**

$$\mathbf{(M1)} 1.5a + 2a + 0.281 + a +$$

[2 marks]

(a.ii)

Write down the mode of X .

[1]

Markscheme

2 (days) **A1**

[1 mark]

(b)

Find the mean of X .

[2]

Markscheme	
using expected value formula (exact) A1 [2 marks]	(M1) (mean =) 2.44 (2.436)

(c)

Identify which one of the following best describes the manager's sampling method.

Circle your answer.

Simple random / Systematic / Convenience / Quota / Stratified

[1]

Markscheme	
convenience (sampling) A1 [1 mark]	

36. 24M.2.SL.TZ1.2

(a)

Find the value of the car at the end of the first year.

[2]

Markscheme	
recognition that a 15 % loss leaves 85 % OR finding 15 % and subtracting from original (\$) 29 750	(M1) $0.85 \times 35\,000$ OR $35\,000 - 0.15 \times 35\,000 =$ A1 Note: Accept (\$) 29 800 .

[2 marks]

(b)

Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar.

[2]

Markscheme

EITHER $29\,750 \times 0.89^9$ **(A1)** **OR** $N = 9$ $I\% = -11$ $PV =$
 $\mp 29\,750$ **(A1)** **THEN** value (FV) = (\$)10 423 **A1** **Note:** For
this **A1** the answer must be rounded to the nearest dollar. Accept (\$)10 441
from using 3 sf answer from part (a).
[2 marks]

(c)

Find the least value of n .

[3]

Markscheme

METHOD 1 attempt to solve the inequality (or equation) $29\,750 \times 0.89^{n-1} <$
 3500 OR table of values **(M1)** 19.3643 ... OR ($n =$
 $19 \Rightarrow$) 3651.80 ... OR ($n = 20 \Rightarrow$) 3250.10 ... **(A1)** **Note:** For
candidates using (\$) 29 800, $n > 19.3787$..., ($n = 19 \Rightarrow$) 3657.93 ..., ($n =$
 $20 \Rightarrow$) 3255.56 $n = 20$ **A1** **METHOD 2** use of the finance app
with $I\% = -11$, $PV = \mp 29\,750$, $FV = \pm 3500$ OR $29\,750 \times 0.89^N <$
 3500 (condone the use of n or x) **(M1)**
($N =$) 18.3643 ... **(A1)** **Note:** For candidates using (\$) 29 800, $N =$
18.3787 ... $n = 20$ **A1**
[3 marks]

37. 24M.2.SL.TZ1.4

(a)

Find the probability that X is more than 1.5 standard deviations above the mean.

[2]

Markscheme	
recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) (M1) $(P(X > 13) =) 0.0668072 \dots = 0.0668$ [2 marks]	A1

(b)

Find the value of k .

[2]

Markscheme	
EITHER equating an appropriate correct normal CDF function to 0.1 or 0.9 (M1) $P(X > 10 + 2k) = 0.1$ OR $P(Z < k) = 0.9$ OR $P(X < 10 - 2k) = 0.1$ OR $P(Z < -k) = 0.1$ OR recognising need to use inverse normal with 0.1 or 0.9 (M1) THEN 1.28155 ... $k = 1.28$ A1 [2 marks]	

38. 24M.2.SL.TZ1.5

(a)

Find the smallest value of t when the particle changes direction.

[2]

Markscheme	
recognition that velocity is zero (M1) $v = 2 \sin(0.5t) + 0.3t - 2 = 0$ $t = 1.68694 \dots t = 1.69$ A1 [2 marks]	

(b)

Find the range of values of t for which the velocity is positive.

[2]

Markscheme

$1.68694 \dots < t < 6.11857 \dots$ $1.69 < t < 6.12$ **A1A1**
Note: Award **A1** for both values, **A1** for correct inequalities.
[2 marks]

(c)

Find the displacement of the particle relative to O when $t = 10$.

[2]

Markscheme

attempt to substitute into the total displacement formula (condone missing or incorrect limits, and absence of dt) **(M1)** $\int_0^{10} (2 \sin(0.5t) + 0.3t - 2) dt$ OR $\int_0^{10} v(t) dt = -2.13464 \dots = -2.13 (m)$ **A1** **Note:** Award **(M1)A0** if -2.13 is followed by 2.13 .
[2 marks]

39. 24M.2.SL.TZ2.2

(a)

Find the value of a and the value of b .

[3]

Markscheme

attempts to find an intersection point **(M1)** $a = -0.916562 \dots$ or $b = 0$
 $a = -0.917, b = 0$ **A1A1**
[3 marks]

(b)

Find the area of the region enclosed by the graphs of f and g .

[3]

Markscheme
<p>let A be the area of the region EITHER attempts to form the required integral involving subtraction (in any order). Accept absence of limits or incorrect limits. Accept absence of dx. (M1) OR shows a graph with the required area shaded (M1) THEN $A = \left(\int_a^b (f(x) - g(x)) dx \right) = \int_{-0.916562\dots}^0 (1 - x^2 - e^{2x}) dx$ (or equivalent) (A1) $A = 0.239855 \dots$ $A = 0.240$ A1 [3 marks]</p>

40. 24M.2.SL.TZ2.3

(a)

Given that $\bar{x} = 7$, verify that $\bar{y} = 16$.

[1]

Markscheme
<p>EITHER $\bar{y} = 2.1875 \times 7 + 0.6875$ A1 OR $\bar{y} = 15.3125 + 0.6875$ A1 THEN $\bar{y} = 16$ AG [1 mark]</p>

(b)

Given that $q - p = 3$, find the value of p and the value of q .

[4]

Markscheme
<p>attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in p and q (M1) $16 = \frac{9+13+p+q+21}{5}$ ($80 = p + q + 43 \Rightarrow p + q = 37$) (A1) attempts to solve two linear equations simultaneously for p and q (one of which is $q = p +$</p>

3) **(M1)** $16 = \frac{9+13+p+p+3+21}{5} (80 = 2p + 46) p = 17$ and $q = 20$ **A1**
[4 marks]

41. 24M.2.SL.TZ2.4

(a)

State the intensity of S_2 .

[1]

Markscheme

$I = 2 \times 10^{-6} \left(= \frac{1}{500\,000} \right)$ (units) **A1**
[1 mark]

(b)

Determine the loudness of S_2 .

[2]

Markscheme

substitutes their doubled I -value from part (a) into L **(M1)** $L = 10 \log_{10} (2 \times 10^{-6} \times 10^{12}) (= 63.0102 \dots) = 63.0$ (decibels) **A1 Note:** Accept $60 + 10 \log_{10} 2$ (decibels) as a final answer. Do not award the final **A1** for $L = 0$ (from $I = 10^{-12}$).
[2 marks]

(c)

Find the corresponding intensity, I , of the thunder.

[3]

Markscheme

$115 = 10 \log_{10} (I \times 10^{12})$ **(A1)** attempts to solve for I **(M1)** $I =$
 $\frac{10^{11.5}}{10^{12}}$ (or equivalent) (= 0.316227 ...) $I = 0.316$ (units) **A1 Note:**
 Accept exact final answers such as $10^{-0.5}$ and $\frac{1}{\sqrt{10}}$.
[3 marks]

42. 24M.2.SL.TZ2.5

(a)

Find the velocity of the particle at $t = 2$.

[1]

Markscheme

$v = -0.996114 \dots v = -0.996$ (ms⁻¹) **A1**
[1 mark]

(b)

Find the maximum velocity of the particle.

[2]

Markscheme

considers $v'(t) = 0$ **(M1)** $t = 0.405833 \dots v_{\max} = 1.18230 \dots v_{\max} =$
 1.18 (ms⁻¹) **A1**
[2 marks]

(c)

Find the acceleration of the particle at the instant it changes direction.

[3]

Markscheme

recognizes that the particle changes direction when $v = 0$ **(M1)** **Note:** Award **(M1)** for $t = 1.65840 \dots$ seen. finds acceleration for their value of t for which $v(t) = 0$ **(M1)** $v'(1.65840 \dots) a = -2.53487 \dots a = -2.53 \text{ (ms}^{-2}\text{)}$ **A1**
[3 marks]

43. 23N.2.SL.TZ1.2

(a)

Find BV .

[2]

Markscheme

$$BV = \sqrt{(8-4)^2 \times (6-3)^2 + (0-10)^2} \quad \text{(A1)} = 11.1803 \dots = 11.2 (= \sqrt{125} = 5\sqrt{5}) \quad \text{(A1)}$$

[2 marks]

(b)

Find the size of $B\hat{V}C$.

[4]

Markscheme

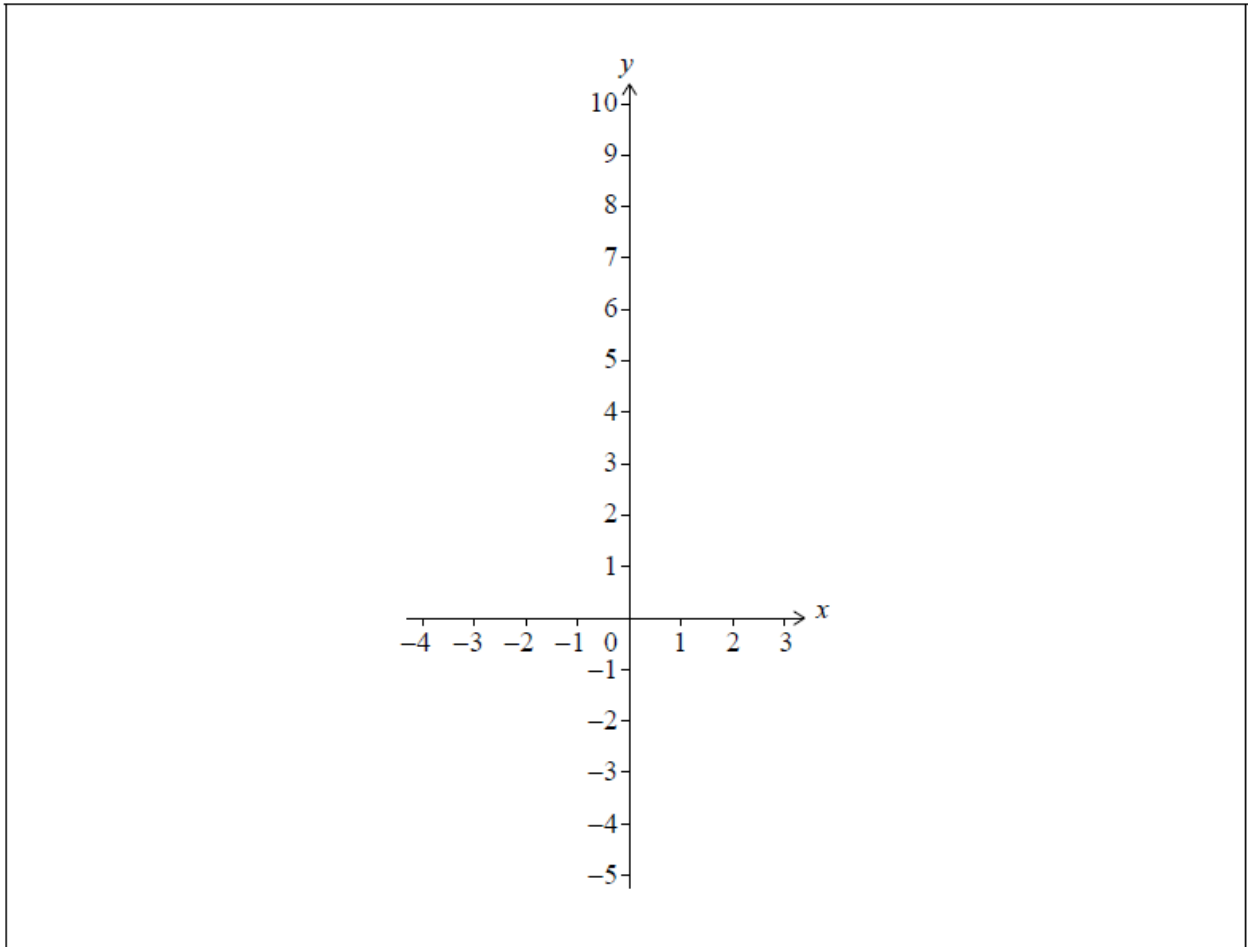
METHOD 1 $BV = VC$ AND $BC = 6$ (seen anywhere) **(A1)**
 attempt to use the cosine rule on triangle BVC for any angle **(M1)** **Note:** Recognition must be shown in context either in terms of labelled sides or side lengths. $\cos B\hat{V}C = \frac{11.1\dots^2 + 11.1\dots^2 - 6^2}{2 \times 11.1\dots \times 11.1\dots}$ OR
 $6^2 = 11.1\dots^2 + 11.1\dots^2 - 2 \times 11.1\dots \times 11.1\dots \cos B\hat{V}C$ **(A1)**
 $B\hat{V}C = 0.543314 \dots B\hat{V}C = 0.543$ (0.542 from 3 sf) (accept 31.1°) **A1** **METHOD 2** let M be the midpoint of BC $BM = 3$ (seen anywhere) **(A1)** attempt to use sine or cosine in triangle BMV or CMV **(M1)** $\arcsin \frac{3}{\sqrt{125}}$ OR $\frac{\pi}{2} - \arccos \frac{3}{\sqrt{125}}$ OR 0.271657... **(A1)** $B\hat{V}C = 0.543314 \dots B\hat{V}C = 0.543$ (0.542 from 3 sf) (accept 31.1°) **A1**
[4 marks]

44. 23N.2.SL.TZ1.3

[[N/A]]

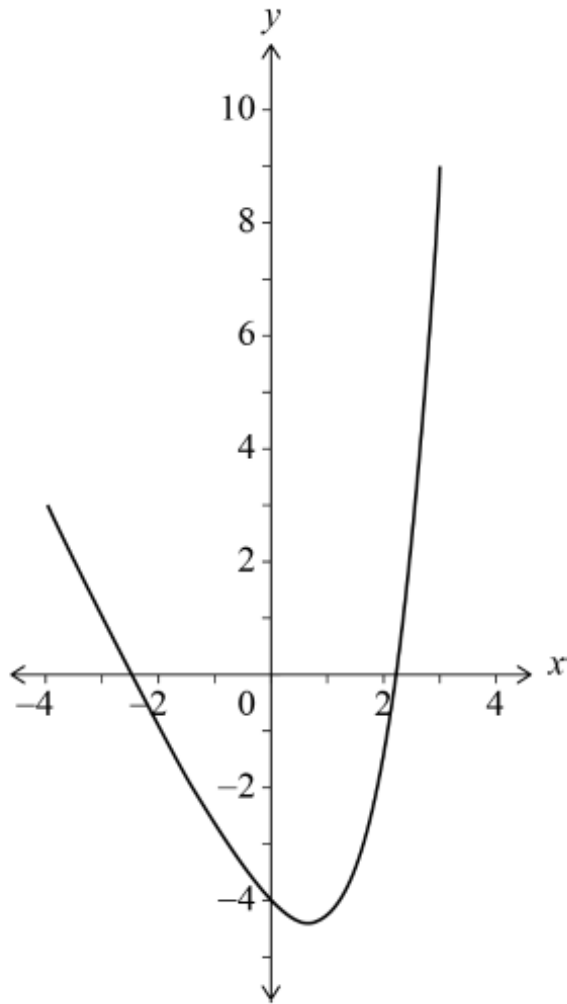
(a)

On the following axes, sketch the graph of f for $-4 \leq x \leq 3$.



[3]

Markscheme



A1A1A1

Note: Award marks as follows:

A1 for approximately correct roots, in the intervals $-3 < x < -2$ and $2 < x < 3$.

A1 for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-4.5 < y < -3.5$, and for local minimum $0.2 < x < 1.2$, $-5 < y < -4$.

A1 for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5$, $2.5 < y < 3.5$ and the right end in the intervals $2.5 < x < 3.5$, $8.5 < y < 9.5$

[3 marks]

(b)

The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k , followed by a vertical translation of c units.

Find the value of k and the value of c .

[2]

Markscheme	
$k = \frac{1}{3}$ down) [2 marks]	A1 $c = -2$ (accept translate/shift 2 (units) A1

45. 23M.2.SL.TZ1.2

(a)

Write down the annual rate of depreciation of the car.

[1]

Markscheme	
9% (accept 0.09)	A1 [1 mark]

(b)

Find the value of the car on 1 January 2028.

[2]

Markscheme		
$t = 5$ (seen anywhere) marks]	(A1) 24961.28 ... 25000 (dollars)	A1 [2

(c)

Find the value of M .

[3]

Markscheme

EITHER $n = 5$ $I\% = 3$ $PV = (\pm)15000$ $P/Y = 1$ $C/Y = 1$ **(A1) Note:**
Award **(A1)** for use of a financial app in their technology with all entries correct. ($\Rightarrow FV = (\pm)17389.11 \dots$)

OR $15\,000 \left(1 + \frac{3}{100}\right)^5 = 17389.11$ **(A1)**

THEN subtracting their value from their answer to part
(b) **(M1)** 7572.17 ... 7570 (dollars) **A1 [3 marks]**

46. 23M.2.SL.TZ1.5

[5]

Markscheme

METHOD 1 $Q_1 = 31.86$ OR $Q_3 = 32.14$ **(A1)** recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) **(M1)** **EITHER** equating an appropriate correct normal CDF function to its correct probability (0.25 or 0.5 or 0.75) **(A2)** OR $z = -0.674489\dots$ OR $z = 0.674489\dots$ (seen anywhere) **(A1)**
 $-0.674489\dots = \frac{31.86-32}{\sigma}$ OR $0.674489\dots = \frac{32.14-32}{\sigma}$ **(A1) THEN**
 $0.207564\dots \sigma = 0.208$ (mm) **A1** **METHOD 2** recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) **(M1)** $z = -0.674489\dots$ OR $z = 0.674489\dots$ **(A1)**
 $(Q_1 =)32 - 0.674489\dots \sigma$ OR $(Q_3 =)32 + 0.674489\dots \sigma$ **(A1)**
 $(Q_3 - Q_1 =)2 \times 0.674489\dots \sigma$ $2 \times 0.674489\dots \sigma = 0.28$ **(A1)**
 $0.207564\dots \sigma = 0.208$ (mm) **A1 [5 marks]**

47. 22N.2.SL.TZ0.3

(a)

Find the coordinates of A.

[2]

Markscheme

(0.708519 ..., 0.639580 ...)
 (0.709, 0.640) ($x = 0.709$, $y = 0.640$) **A1A1**

[2 marks]

(b)

Find the x -coordinate of B.

[1]

Markscheme

$x = 1.10$ accept (1.10,0) 1.09885 ...
A1

[1 mark]

(c)

Find the total area enclosed by the graph of f , the x -axis and the line $x = 2$.

[3]

Markscheme

METHOD 1

$\int_0^2 |f(x)| dx$ **(A1)** 4.61117 ...
 area = 4.61 **A2**

METHOD 2

$-\int_{1.09885\dots}^2 f(x) dx$ OR $\int_{1.09885\dots}^2 |f(x)| dx$ OR 4.17527 ... **(A1)**
 $\int_0^{1.09885\dots} f(x) dx - \int_{1.09885\dots}^2 f(x) dx$ OR 0.435901 ... +
 4.17527 ... **(A1)** 4.61117 ...
 area = 4.61 **A1**

[3 marks]

