

1. 25M.1.SL.TZ1.1

Write each of the following expressions in the form $\ln k$, where $k \in \mathbb{Z}^+$.

(a) $\ln 3 + \ln 4$ [1]

(b) $3\ln 2$. [2]

(c) $-\ln \frac{1}{2}$ [2]

2. 25M.1.SL.TZ1.2

Consider the function $f(x) = \frac{4x^3}{3} - 16x$, where $x \in \mathbb{R}$.

The graph of $y = f(x)$ has a local minimum point at (p, q) where $p > 0$.

Find the value of p and the value of q . [5]

3. 25M.1.SL.TZ1.3

Bob invests 1 000 dinar in an account which pays a nominal annual interest rate of 4 % compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1\,000(1 + k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k . [1]

(b) Expand and simplify $(1 + x)^4$. [2]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

4. 25M.1.SL.TZ1.4

Find the area completely enclosed by the curves $y = e^x$, $y = -e^x$, and the lines $x = -1$ and $x = 1$. [4]

5. 25M.1.SL.TZ1.5

Consider events A and B such that $P(A') = P(A \cup B) = \frac{3}{4}$ and $P(B|A) = \frac{2}{3}$.

(a) Find $P(A \cap B)$. [3]

(b) Show that events A and B are independent. [3]

6. 25M.1.SL.TZ2.1

Consider the function $f(x) = x^3 + 5x^2 - 8$, where $x \in \mathbb{R}$.

(a) Find $f'(1)$. [2]

(b) Find the equation of the tangent to the graph of f at $x = 1$. [2]

7. 25M.1.SL.TZ2.2

The derivative of a function g is given by $g'(x) = \cos x + e^{2x}$, where $x \in \mathbb{R}$.

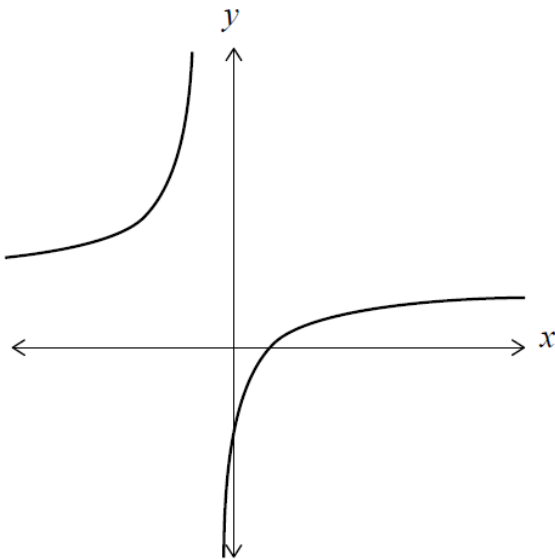
Given that $g(0) = 7$, find $g(x)$.

[5]

8. 25M.1.SL.TZ2.3

The function f is defined by $f(x) = \frac{3x-2}{2x+1}$ for $x \in \mathbb{R}, x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.



(a) Write down the value of $f(0)$.

[1]

(b) Write down the equation of the horizontal asymptote.

[1]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

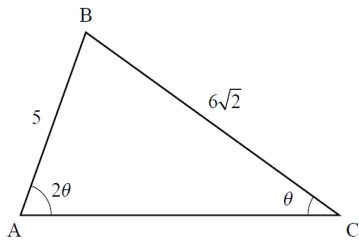
(c) Find the range of g .

[3]

9. 25M.1.SL.TZ2.4

The following diagram shows a non-right angled triangle ABC .

diagram not to scale



$AB = 5$, $BC = 6\sqrt{2}$, $\hat{A}CB = \theta$ and $\hat{B}AC = 2\theta$, where $0 < \theta < \frac{\pi}{2}$.

(a) Using the sine rule, show that $\cos \theta = \frac{3\sqrt{2}}{5}$. [3]

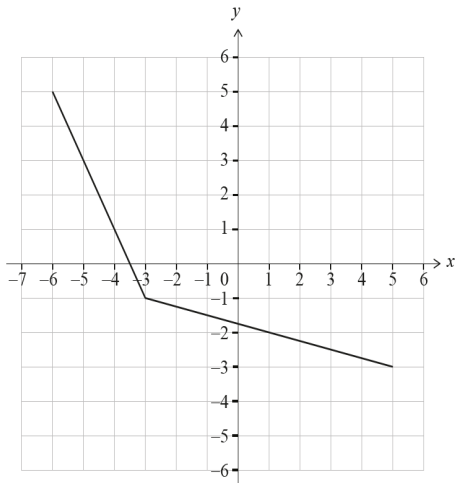
(b) Hence, find $\sin \theta$. [2]

Point D is located on $[AC]$ such that the area of triangle BCD is $2\sqrt{14}$.

(c) Find DC . [2]

10. 25M.1.SL.TZ3.3

The following diagram shows the graph of $y = f(x)$, for $-6 \leq x \leq 5$.



- (a) Write down the value of $f(-3)$. [1]
- (b) State the domain of f^{-1} , the inverse function of f . [1]
- (c) Find the value of x that satisfies $f^{-1}(2x - 7) = -3$. [3]

11. 24N.1.SL.TZ1.1

Consider the following set of ordered data.

27	27	31	32	33	37	41	46	54	57
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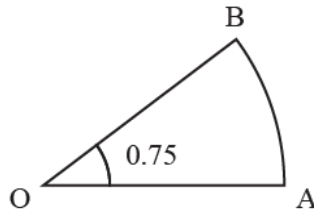
- (a) Write down
- (a.i) the mode; [1]
- (a.ii) the range; [1]
- (a.iii) the median. [1]
- (b) Find the interquartile range. [3]

12. 24N.1.SL.TZ1.2

Points A and B lie on a circle with centre O and radius r cm, where $\widehat{AOB} = 0.75$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is 6 cm^2 .

(a) Find the value of r . [3]

(b) Hence, find the perimeter of sector OAB. [2]

13. 24N.1.SL.TZ1.4

Prove that $(2n + 5)^2 - (2n - 5)^2$ is a multiple of 10 for all $n \in \mathbb{Z}^+$. [4]

14. 24M.1.SL.TZ1.1

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

(b) Find an expression for u_n , the n th term. [2]

15. 24M.1.SL.TZ1.2

Claire rolls a six-sided die 16 times.

The scores obtained are shown in the following frequency table.

Score	Frequency
1	p
2	q
3	4
4	2
5	0
6	3

It is given that the mean score is 3.

(a) Find the value of p and the value of q .

[5]

Each of Claire's scores is multiplied by 10 in order to determine the final score for a game she is playing.

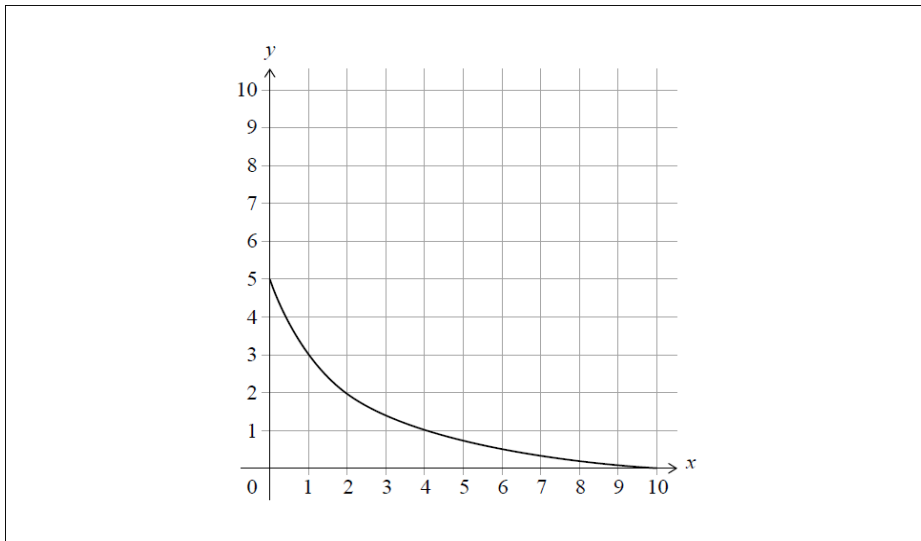
(b) Write down the mean final score.

[1]

16. 24M.1.SL.TZ2.1

The graph of $y = f(x)$ for $0 \leq x \leq 10$ is shown in the following diagram.

The graph intercepts the axes at $(10, 0)$ and $(0, 5)$.



(a) Write down the value of

(a.i) $f(4)$; [1]

(a.ii) $f \circ f(4)$; [1]

(a.iii) $f^{-1}(3)$. [1]

(b) On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes. [2]

17. 24M.1.SL.TZ2.3

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

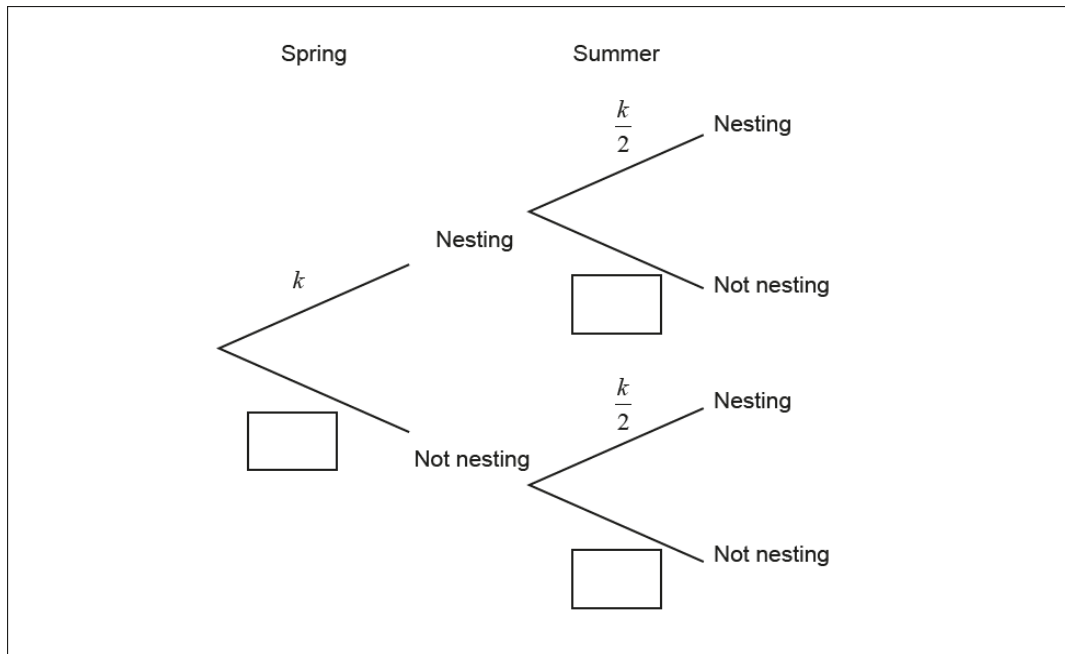
18. 24M.1.SL.TZ2.5

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k .

The probability of nesting in Summer is $\frac{k}{2}$.

This is shown in the following tree diagram.



(a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k . [2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\frac{5}{9}$.

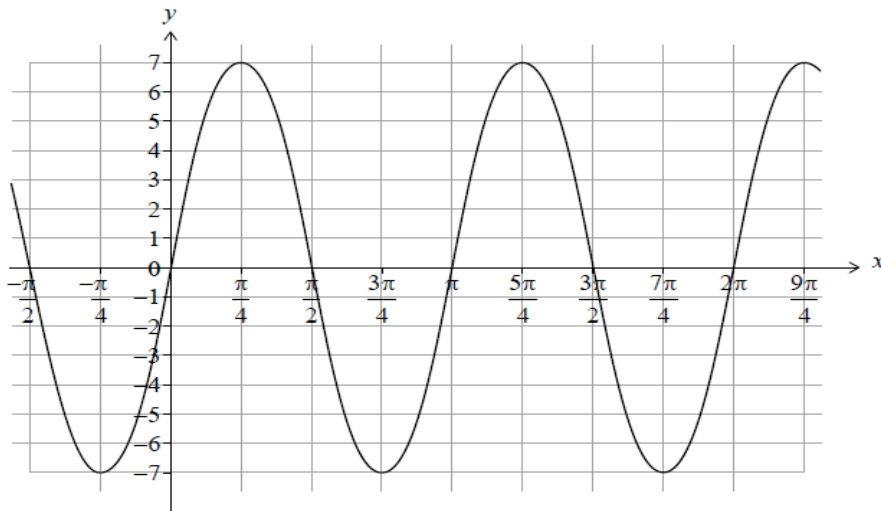
(b.i) Show that $9k^2 - 27k + 8 = 0$. [3]

(b.ii) Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution. [1]

19. 23N.1.SL.TZ1.1

Consider the function $f(x) = a \sin (bx)$ with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f .



- (a) Write down the value of a . [1]
- (b.i) Write down the period of f . [1]
- (b.ii) Hence, find the value of b . [2]
- (c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

20. 23N.1.SL.TZ1.3

Events A and B are such that $P(A) = 0.7$, $P(B) = 0.75$ and $P(A \cap B) = 0.55$.

- (a) Find $P(A \cup B)$. [2]
- (b) Hence, otherwise find $P(A' \cap B')$. [2]

21. 23M.1.SL.TZ1.1

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of $[PQ]$.

(a) Find the coordinates of M . [2]

Line L is perpendicular to $[PQ]$ and passes through M .

(b) Find the gradient of L . [2]

(c) Hence, write down the equation of L . [1]

22. 23M.1.SL.TZ1.2

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

(b.ii) the horizontal asymptote. [1]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

23. 23M.1.SL.TZ1.4

(a) Show that the equation $\cos 2x = \sin x$ can be written in the form $2 \sin^2 x + \sin x - 1 = 0$. [1]

(b) Hence, solve $\cos 2x = \sin x$, where $-\pi \leq x \leq \pi$. [5]

24. 23M.1.SL.TZ1.5

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^x$ has at least one real solution. [6]

25. 22M.1.SL.TZ1.2

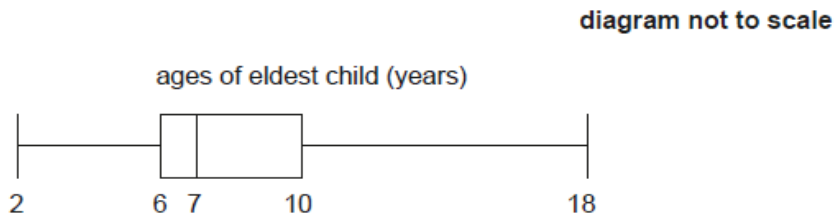
(a) The expression $\frac{3\sqrt{x}-5}{\sqrt{x}}$ can be written as $3 - 5x^p$. Write down the value of p . [1]

(b) Hence, find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}} \right) dx$. [4]

26. 22M.1.SL.TZ1.3

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.



(a) Find the largest value of c that would not be considered an outlier. [3]

The regression line of a on c is $a = \frac{7}{4}c + 20$. The regression line of c on a is $c = \frac{1}{2}a - 9$.

(b.i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child. [2]

(b.ii) Find the mean age of all the adults surveyed. [2]

27. 22M.1.SL.TZ1.5

Consider the curve with equation $y = (2x - 1)e^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$.

The tangent to the curve at the point where $x = 1$ is parallel to the line $y = 5e^kx$.

Find the value of k . [5]

28. 25M.2.SL.TZ1.1

Consider the function $f(x) = \frac{(x-1)^2}{x}$, where $x \in \mathbb{R}, x \neq 0$.

(a) Show that $\frac{(x-1)^2}{x} = x - 2 + \frac{1}{x}$. [2]

(b) Hence, find $\int f(x) dx$. [3]

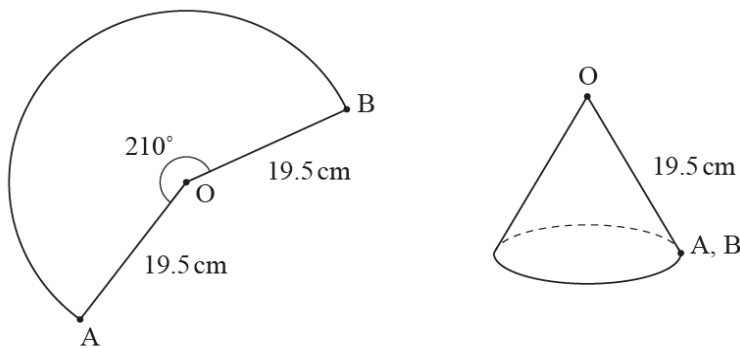
29. 25M.2.SL.TZ1.2

The points A and B lie on a circle, with centre O and radius 19.5 cm , such that $\widehat{BOA} = 210^\circ$.

A piece of paper is cut into the shape of the sector BOA .

A hollow cone with no base is constructed from the sector by joining the points A and B .
The sector forms the curved surface of the cone.

This is shown in the following diagrams.



Find

(a) the area of the sector BOA ; [3]

(b) the radius of the cone. [3]

30. 25M.2.SL.TZ1.3

Consider the function $f(x) = a \tan(2x) + b$, where $x \neq \frac{(2n+1)\pi}{4}$, $n \in \mathbb{Z}$ and $a, b \in \mathbb{R}$.

(a) Write down the period of f . [1]

The graph of $y = f(x)$ passes through the points $(\frac{\pi}{12}, 5)$ and $(\frac{\pi}{3}, 7)$.

(b) Find the value of a and the value of b . [3]

31. 25M.2.SL.TZ1.4

A population, P , has a rate of change modelled by $\frac{dP}{dt} = -104\,000e^{-0.0145t}$, where t is the time measured in years since the **start** of 2022.

At the start of 2022, the population was 6.78×10^6 .

Based on this model, find the predicted population at the start of 2026. [6]

32. 24N.2.SL.TZ1.1

Consider the function $f(x) = 7\sqrt{x} - x - 7$, where $0 \leq x \leq 60$

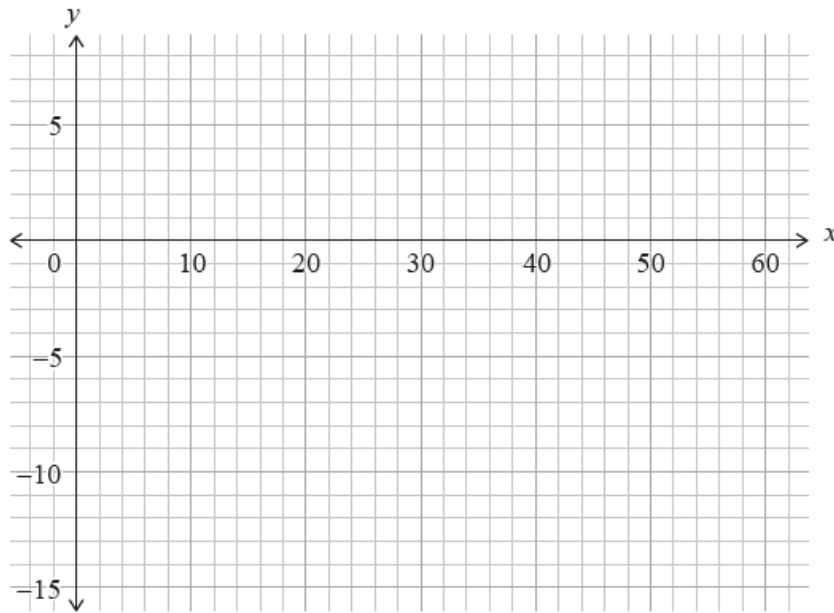
(a) Find the value of

(a.i) $f(0)$; [1]

(a.ii) $f(60)$. [1]

(b) Find the two roots of $f(x) = 0$. [2]

(b) Sketch the graph of $y = f(x)$ on the following grid.



[3]

33. 24N.2.SL.TZ1.2

Find the coefficient of x^8 in the expansion of $(2x - 5)^{11}$.

[4]

34. 24N.2.SL.TZ1.4

A discrete random variable, X , has the following probability distribution:

$$P(X = x) = \frac{kx}{15} \text{ for } x \in \{2, 4, 7, 10\}.$$

(a) Find the value of k .

[2]

(b) Find $E(X)$.

[3]

35. 25M.2.SL.TZ3.4

A supermarket analyses the shopping habits of its customers.

The number of times, X , each customer visits the supermarket in a week is given by the following probability distribution.

x	1	2	3	4	5	≥ 6
$P(X=x)$	$1.5a$	$2a$	0.281	a	0.026	0

(a.i) Find the value of a . [2]

(a.ii) Write down the mode of X . [1]

(b) Find the mean of X . [2]

The manager wants to know why customers come to their supermarket. They survey the first 50 customers to arrive at the supermarket on a particular day.

(c) Identify which one of the following best describes the manager's sampling method.

Circle your answer.

Simple random / Systematic / Convenience / Quota / Stratified

[1]

36. 24M.2.SL.TZ1.2

Darren buys a car for \$35 000. The value of the car decreases by 15 % in the first year.

(a) Find the value of the car at the end of the first year. [2]

After the first year, the value of the car decreases by 11 % in each subsequent year.

(b) Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar. [2]

When Darren has owned the car for n complete years, the value of the car is less than 10 % of its original value.

(c) Find the least value of n . [3]

37. 24M.2.SL.TZ1.4

The random variable X is normally distributed with mean 10 and standard deviation 2.

(a) Find the probability that X is more than 1.5 standard deviations above the mean. [2]

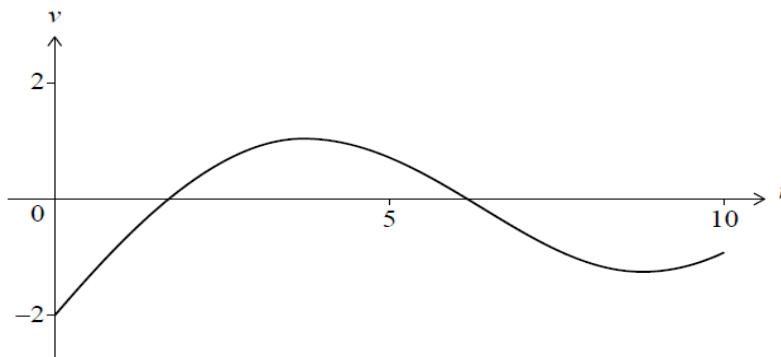
The probability that X is more than k standard deviations above the mean is 0.1, where $k \in \mathbb{R}$.

(b) Find the value of k . [2]

38. 24M.2.SL.TZ1.5

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O . For $0 \leq t \leq 10$ its velocity, v metres per second, is given by $v = 2 \sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.



(a) Find the smallest value of t when the particle changes direction. [2]

The displacement of the particle is measured in metres from O .

(b) Find the range of values of t for which the velocity is positive. [2]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

39. 24M.2.SL.TZ2.2

The functions f and g are both defined for $-1 \leq x \leq 0$ by

$$f(x) = 1 - x^2$$

$$g(x) = e^{2x}.$$

The graphs of f and g intersect at $x = a$ and $x = b$, where $a < b$.

(a) Find the value of a and the value of b . [3]

(b) Find the area of the region enclosed by the graphs of f and g . [3]

40. 24M.2.SL.TZ2.3

Consider the following bivariate data set where $p, q \in \mathbb{Z}^+$.

x	5	6	6	8	10
y	9	13	p	q	21

The regression line of y on x has equation $y = 2.1875x + 0.6875$.

The regression line passes through the mean point (\bar{x}, \bar{y}) .

(a) Given that $\bar{x} = 7$, verify that $\bar{y} = 16$. [1]

(b) Given that $q - p = 3$, find the value of p and the value of q . [4]

41. 24M.2.SL.TZ2.4

The loudness of a sound, L , measured in decibels, is related to its intensity, I units, by $L = 10\log_{10}(I \times 10^{12})$.

Consider two sounds, S_1 and S_2 .

S_1 has an intensity of 10^{-6} units and a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1 .

(a) State the intensity of S_2 . [1]

(b) Determine the loudness of S_2 . [2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I , of the thunder. [3]

42. 24M.2.SL.TZ2.5

A particle moves in a straight line such that its velocity, $v \text{ ms}^{-1}$, at time t seconds is given by $v(t) = 1 + e^{-t} - e^{-\sin 2t}$ for $0 \leq t \leq 2$.

(a) Find the velocity of the particle at $t = 2$. [1]

(b) Find the maximum velocity of the particle. [2]

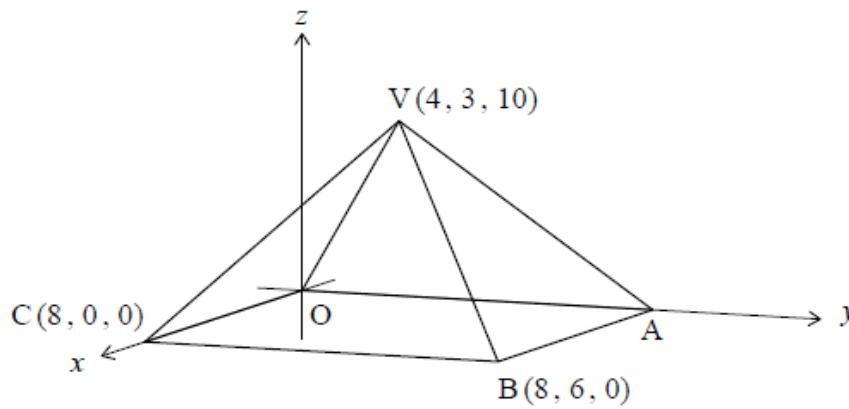
(c) Find the acceleration of the particle at the instant it changes direction. [3]

43. 23N.2.SL.TZ1.2

The following diagram shows a pyramid with vertex V and rectangular base of $OABC$.

Point B has coordinates $(8, 6, 0)$, point C has coordinates of $(8, 0, 0)$ and point V has coordinates $(4, 3, 10)$.

diagram not to scale



(a) Find BV .

[2]

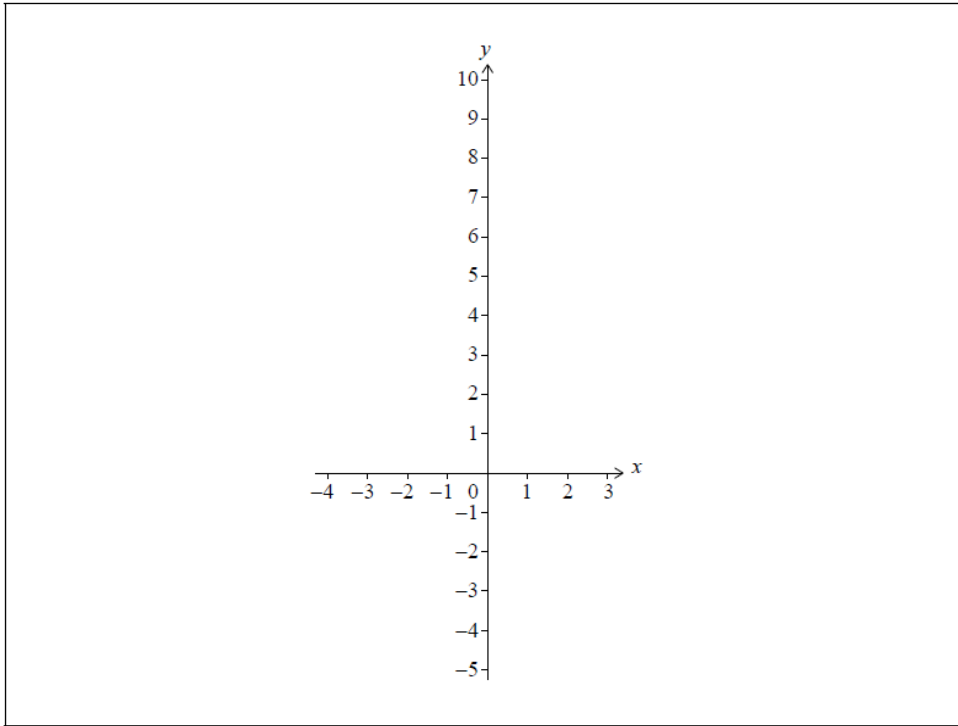
(b) Find the size of $B\hat{V}C$.

[4]

44. 23N.2.SL.TZ1.3

Consider the function $f(x) = e^x - 2x - 5$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$.



[3]

The function g is defined by $g(x) = e^{3x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k , followed by a vertical translation of c units.

Find the value of k and the value of c .

[2]

45. 23M.2.SL.TZ1.2

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in *USD* (\$).

(a) Write down the annual rate of depreciation of the car. [1]

(b) Find the value of the car on 1 January 2028. [2]

Alvie wants to buy this car. On 1 January 2023, he invested \$15 000 in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra \$ M .

(c) Find the value of M . [3]

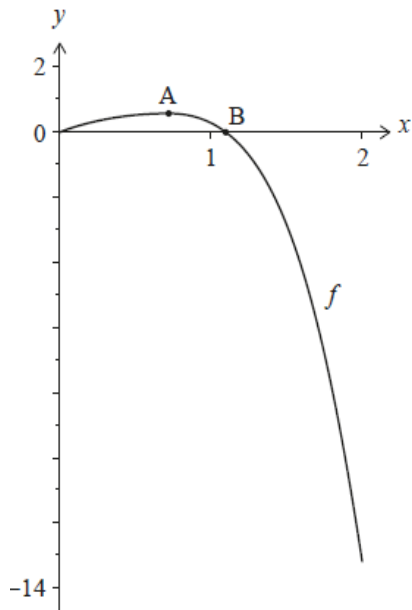
46. 23M.2.SL.TZ1.5

A company manufactures metal tubes for bicycle frames. The diameters of the tubes, D mm, are normally distributed with mean 32 and standard deviation σ . The interquartile range of the diameters is 0.28.

Find the value of σ . [5]

47. 22N.2.SL.TZ0.3

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \leq x \leq 2$. The graph of f is shown in the following diagram.



The graph of f has a local maximum at point A. The graph intersects the x -axis at the origin and at point B.

- (a) Find the coordinates of A. [2]
- (b) Find the x -coordinate of B. [1]
- (c) Find the total area enclosed by the graph of f , the x -axis and the line $x = 2$. [3]