Mathematics
Standard Level
Paper 2

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

An amount of $1000 is invested into a bank account at an interest rate of 1.5%, compounded annually.

(a) Calculate the amount of money in the account after four years. [3]

(b) Determine the number of years, n, after which the account first exceeds $1300, where n is an integer. [3]

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2. [Maximum mark: 6]

A study is conducted to compare the monthly e-commerce sales of nine separate online stores to their monthly online advertising costs. The table below shows the monthly e-commerce sales (y) in 1000$ of each online store and their monthly online advertising costs (x) in 1000$.

The relationship between the monthly e-commerce sales and the monthly online advertising costs can be modelled by the regression line with equation $y = ax + b$.

Online Advertising Costs (x)	1.4	1.7	2.3	1.1	4.7	2.2	2.9	3.8	1.9
E-Commerce Sales (y)	343	371	587	320	921	492	646	835	413

(a) (i) Find Pearson's product moment correlation coefficient, r.

(ii) Write down the value of a and the value of b. [3]

One of these nine online stores decides to increase their budget for monthly online advertising costs by $500.

(b) Based on the given data, determine how the store's monthly e-commerce sales could be expected to alter. [2]

An online store separate from the study has monthly online advertising costs of $7000.

(c) Comment on the appropriateness of using your regression line to predict the monthly e-commerce sales of this separate online store. [1]

(This question continues on the following page)

(Question 2 continued)

3. [Maximum mark: 6]

Triangle FGH has $FG = 8$ cm, $GH = 9$ cm and area 24 cm^2.

(a) Find $\sin \hat{G}$. [2]

(b) Hence, find the two possible values of FH, giving your answers correct to two decimal places. [4]

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4. [Maximum mark: 6]

The sum of the first n terms of a series is given by

$$S_n = 3n^2 + n, \ n \in \mathbb{Z}^+$$

(a) Find the first three terms of the series. [3]

(b) Find an expression for the n^{th} term of the series, giving your answer in terms of n. [3]

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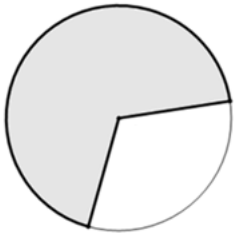
5. [Maximum mark: 6]

Find $\int x\left(x^2+1\right)^3 \mathrm{d}x$.

6. [Maximum mark: 6]

In the figure below, the shaded sector has a perimeter that is equal to the circumference of the circle. The location of a point inside the circle is chosen at random. Find the probability that the randomly chosen point is located inside the shaded sector.



Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 13]

Consider the function g defined as $g(x) = \frac{x}{3-x}$, $x \neq 3$.

(a) (i) Show that the inverse of g is $g^{-1}(x) = \frac{3x}{x+1}$.

(ii) State the domain and range of g^{-1}. [4]

(b) (i) Sketch the graph of g^{-1} for $-5 \leq x \leq 5$ and $-4 \leq y \leq 8$, including all asymptotes.

(ii) Write down the equations of the asymptotes.

(iii) Write down the x-intercept of the graph of g^{-1}. [7]

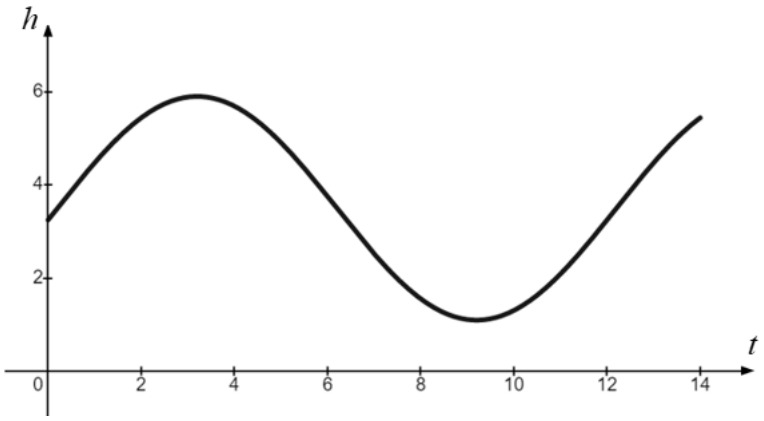
(c) Find the area of the region enclosed by the graph of g^{-1}, the x-axis and the line $x = 4$. [2]

Do **not** write solutions on this page.

8. [Maximum mark: 16]

The height, in metres, of the tide in a bay is modelled by the function $h(t) = a\cos(b(t-c)) + d$, where t is the number of hours after midnight, and a, b, c and d are positive constants.

The graph below shows the height of the water for $0 \le t \le 14$.



The first high tide (maximum height) occurs at 03:12 and the next high tide occurs 12 hours later. The height of the tide ranges from a low tide (minimum) of 1.1 metres and a high tide (maximum) of 5.9 metres.

(a) Show that $b = \frac{\pi}{6}$. [2]

(b) Find the value of a and the value of d. [4]

(c) Find the smallest value of c, where $c > 0$. [3]

(d) Find the height of the water at: [4]

(i) 00:00;

(ii) 08:00.

(e) Determine the number of hours, over a 24-hour period, for which the tide is lower than 3 metres. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

It has been determined that the volume of fluid in a bottle of olive oil filled by a robotic dispenser in a factory is normally distributed with a mean of 748 ml and a standard deviation of 2.4 ml.

(a) Find the probability that a randomly selected bottle of olive oil from the factory contains more than 750 ml. [2]

(b) The amount of olive oil is measured for each bottle in a random sample of 12 bottles. Find the probability that exactly 4 of them contain more than 750 ml. [3]

(c) Find the minimum number of bottles that would need to be sampled so that the probability of getting at least one bottle containing more than 750 ml of olive oil is greater than 0.98. [3]

The same factory produces packages of olives, such that the weight, A grams, of olives in a package is normally distributed with mean μ grams and standard deviation σ grams.

(d) Given that $\mathrm{P}(A < 850) = 0.09$ and $\mathrm{P}(A < 900) = 0.97$, find the value of μ and the value of σ. [7]